Residual-based tests for cointegration and multiple regime shifts

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Abstract

In this paper we examine the properties of several cointegration tests when long run parameters are subject to multiple shifts, resorting to Monte Carlo methods. We assume that the changes in cointegration regimes are governed by a unobserved Markov chain process. This specification has the considerable advantage of allowing for an unspecified number of stochastic breaks, unlike previous works that consider a single, deterministic break. Our Monte Carlo analysis reveals that testing cointegration with the usual procedures is a quite unreliable task, since the performance of the tests is poor for a number of plausible regime shifts parameterizations.

Key Words: Cointegration; Tests; Structural change; Markov Switching; Monte Carlo

JEL Classification: C12; C22; C52

1 Introduction

The concept of cointegration has dominated the debate in time series econometrics in the past decade, by stressing the possible existence of long run equilibrium relationships among non-stationary variables. More recently, researchers became concerned with the effects that structural changes may have on econometric models. Indeed, failure to detect and account for parameter shifts is a serious form of misspecification, therefore affecting inference and leading to poor forecasting performances (see Clements and Hendry, 1999, for example). This is especially relevant for cointegration analysis, since it normally involves long time spans of data, which, consequently, are likely to display structural breaks. Several papers deal with this possibility in a number of empirical applications, such as money demand, term structure of interest rates, purchasing power parity, among others (see references below).

Therefore, it is natural to ask what is the impact of possible multiple parameter changes on the finite-sample power and size properties of several cointegration tests. In this paper, we investigate this issue in a single-equation framework, resorting to Monte Carlo methods. We assume that distinct cointegration regimes may exist, in which the shifts are governed by an unobserved Markov chain process. This specification has the considerable advantage of allowing for an unspecified number of endogenous, stochastic breaks, unlike previous works that either consider a single, deterministic break or assume that the break points are known when cointegration is being tested.

We also analyse the implications of changing variances in the error process, an issue that was not considered in previous literature. Furthermore, we study the properties of distinct cointegration tests. Besides the "classical" Augmented Dickey-Fuller (ADF) test, we analyse the performance of Phillips-Ouliaris tests and of cointegration tests developed by Gregory and Hansen (1996). The latter are conceived to be robust to regime shifts in the cointegration vector. It is natural, thus, to question how robust will they be to Markov regime switches. On the other hand, we also look at the behaviour of the KPSS-type test for the null hypothesis of cointegration derived by McCabe, Leybourne and Shin (1997). The robust-to-breaks test for the null of cointegration suggested by Hao (1996) is also considered.

Markov switching models have been extensively (and successfully) used to characterize and account for regime changes that typically occur in economic and financial time series, such as GDP, stock prices, interest rates, inflation rates or exchange rates, for example (see Kim and Nelson, 1999 for a survey). Given their flexibility, it would be natural to extend their use to model changes in long run relationships. Hall, Psaradakis and Sola (1997) and Krolzig (1997), for example, illustrate the usefulness of such a specification by analysing the Japanese consumption function and co-movements in international business cycles, respectively. Nevertheless, none of the papers analyses explicitly the effects of Markov-type of changes on cointegration tests.

Since the seminal work of Perron (1989), it is known from the literature on unit roots and structural breaks (see Maddala and Kim, 1998 and Stock, 1994 for surveys) that unit root tests have difficulties (i.e. low power) in distinguishing between an $I(1)$ series and $I(0)$ processes with breaks. Conversely, Leybourne, Mills and Newbold (1998) and Leybourne and Newbold (2000) demonstrate that the routine application of the Dickey-Fuller test when the true process is $I(1)$ with a relatively early break leads to more frequent rejections of the null of a unit root. However, Lee (2000) disputes this result, arguing that it is due to efficiency losses, given that the first observation is discarded when computing the DF test. Tests using the full unconditional likelihood will not suffer from this "spurious stationarity" phenomenon.

On the other hand, the implications of breaks for the performance of stationarity tests was studied by Lee, Huang and Shin (1997). They show that these tests, when used ignoring an existing break in a stationary process, will be biased towards rejecting the null of stationarity in favour of the false alternative of a unit root. Notwithstanding this, there will be no power losses if the unit root alternative is true, since the limiting distribution is asymptotically invariant to this type of shifts. We argue in this paper that this overrejection problem may be related with the fact that some stationarity tests, due to the way they are constructed, also have power against structural change.

Concerning the effects of changes in variance, Hamori and Tokihisa (1997) show that spurious stationarity will also arise if DF tests are applied to a process that suffered an upward break in variance. Early shifts will contribute to increase the size distortions and the effects do not seem to disappear asymptotically. On the other hand, Kim, Leybourne and Newbold (2000) consider the case of a decrease in variance. Unlike what was conjectured by Hamori and Tokihisa (1997), severe spurious rejections occur in this situation, since these authors restricted their analysis to the simple model with no constant and no trend.

More recently, two related papers by Nelson, Piger and Zivot (2001) and Psaradakis (2001) appeared, examining the behaviour of unit root tests when time series are subject to Markov parameter changes. These studies show that, in general, both standard unit root tests and single-break robust tests will do a poor job. These papers generalize the results in Franses and Haldrup (1994), who found that the presence of large and frequent (additive) outliers leads to severe overrejections by unit root and cointegration tests. Our study may be viewed as an extension of these papers concerning cointegration issues.

Previous literature on structural change and cointegration has focused on developing procedures to detect breaks or to estimate the temporal location of eventual shifts. Papers addressing these issues include Hansen (1992), Quintos and Phillips (1993), Hao (1996), Andrews, Ploberger and Lee (1996), Bai, Lumsdaine and Stock (1998), Seo (1998) and Kuo (1998), among others (see also Maddala and Kim, 1998 for a general survey). However, these procedures will in general only be valid if the variables are in fact cointegrated. This adds importance to the understanding of the properties of cointegration tests when parameter changes take place.

While there is a vast literature on the impact of structural breaks on univariate time series, papers specifically dealing with the effect of parameter non-constancy on cointegration tests are less abundant. These include the work of Gregory, Nason and Watt (1996), who, in the context of the linear quadratic model, found that the ADF test has its power considerably decreased in the presence of a structural break. This is not necessarily a weakness, since the alternative of Engle-Granger cointegration implies an invariant relationship. Little is said about possible size distortions. These conclusions are also supported by Gregory and Hansen (1996). On the other hand, Campos, Ericsson and Hendry (1996) analyse cointegration tests when the marginal process of one of the cointegrating regressors is stationary with a break, confirming the decrease in power of the ADF test. It should be noted, however, that these studies are limited in scope, in the sense that they only address one type of structural break (fixed, single deterministic shift) and concentrate on the properties of the ADF cointegration test. Moreover, it remains an open question whether the results from this literature are general enough to encompass regime shifts as specified in this paper, both in terms of finite-sample power and size.

Thus, our paper extends and unifies existing studies focusing on structural change and cointegration. Moreover, our analysis stresses parameter non-constancy that is empirically plausible and economically meaningful in this context. To illustrate the problem, we reestimate the present value model with Markov switching of Driffill and Sola (1998) and look at the performance of cointegration tests.

The paper proceeds as follows. The next section reviews the cointegration tests of interest. Section 3 describes the experimental design of our simulations, while Section 4 reports and discusses the results of the experiments. Section 5 provides an empirical illustration of the problem using US data on stock prices and dividends and Section 6 concludes.

2 Cointegration Tests

In this section, we provide a necessarily brief description of the cointegration tests examined in the subsequent Monte Carlo study. Given the model

$$y_t = \alpha + \beta' x_t + u_t, \tag{1}$$

where $z_t = (y_t, x_t)$ is a $(k+1) \times 1$ vector of $I(1)$ variables, x_t possibly containing deterministic elements (such as a time trend), the variables in z_t will be cointegrated if u_t is stationary. To test this hypothesis in this paper, we employ "standard" tests with the null hypothesis of no cointegration, tests which have cointegration as their null, as well as cointegration tests allowing for regime shifts.

2.1 Standard Cointegration Tests

The ADF and the Z_α and $Z_t$tests of Phillips and Ouliaris (1990) are among the most popular cointegration tests, having been extensively used and discussed in the literature. They may be viewed as an application of their unit root counterparts to test whether the residuals $\hat{u}_t$ from (1) have a unit root or, by contrast, are stationary. While the ADF test corrects for serial correlation by adding lagged $\Delta\hat{u}_t$ terms in the test regression $\Delta\hat{u}_t = (\rho - 1)\hat{u}_{t-1} + \eta_t$, Phillips-Ouliaris tests make use of a nonparametric modification, which involves the estimation of σ^2_η, the long run variance of the errors η_t.

To select an appropriate lag length for the ADF test, we adopt a t-test downward selection procedure, by setting the maximum lag equal to 6 and then testing downward until a significant last lag is found, at the 5% level. Finite-sample critical values computed as in MacKinnon (1991) will be used in our experiments. Turning to Z_α and Z_t tests, the long run variance σ^2_η is estimated by means of a prewhitened quadratic spectral kernel with an automatically selected bandwidth estimator, using a first-order autoregression as a prewhitening filter, as recommended in Andrews and Monahan (1992).

2.2 Gregory-Hansen Tests

Gregory and Hansen (1996), building upon Zivot and Andrews (1992), generalized the standard cointegration tests by considering an alternative hypothesis in which the cointegration vector may suffer a regime shift at an unknown timing. They analyzed models that accommodate under the alternative the possibility of changes in parameters, namely a level shift model (C), a model with a level shift plus trend (C/T), a "regime shift" model (C/S) where both the constant and slope parameters change, as well as a regime shift model where a trend shift is added ($C/S/T$),

$$\begin{aligned}
y_t &= \mu_1 + \mu_2 D_t + \beta' x_t + u_t, \qquad t = 1, ..., T, && (C)\\
y_t &= \mu_1 + \mu_2 D_t + \alpha t + \beta' x_t + u_t, && (C/T)\\
y_t &= \mu_1 + \mu_2 D_t + \beta_1' x_t + \beta_2' x_t D_t + u_t, && (C/S)\\
y_t &= \mu_1 + \mu_2 D_t + \alpha_1 t + \alpha_2 t D_t + \beta_1' x_t + \beta_2' x_t D_t + u_t. && (C/S/T)
\end{aligned}$$

The vector x_t of $I(1)$ variables is of dimension k, u_t should be a stationary disturbance and D_t is a dummy variable of the type

$$D_t = \begin{cases} 0, \text{ if } t > [T\tau] \\ 1, \text{ if } t \leq [T\tau]. \end{cases} \tag{2}$$

Here, $\tau \in J$ denotes the unknown relative timing of the break point and [.] denotes the integer part operator. The trimming region defined by J may be any compact set of $(0, 1)$, but following earlier literature, Gregory and Hansen (1996) propose $J = (0.15, 0.85)$.

As with the previous tests, these are residual-based cointegration tests that evaluate if the error term is $I(1)$ under the null. In this framework, however, since the change point or its occurrence are unknown, the testing procedures involve computing the usual statistics for all possible break points $\tau \in J$ and then selecting the smallest value obtained, since it will potentially present greater evidence against the null hypothesis of no cointegration. Therefore, one should observe the values of

$$GH\text{-}Z_\alpha = \inf_{\tau \in J} Z_\alpha, \tag{3}$$

$$GH\text{-}Z_t = \inf_{\tau \in J} Z_t, \tag{4}$$

$$GH\text{-}ADF = \inf_{\tau \in J} ADF. \tag{5}$$

Nevertheless, as pointed out by the authors, these tests possess power against other alternatives, namely "stable" cointegration. Hence, a rejection of the null hypothesis does not necessarily imply changes in the cointegration vector, since an invariant relationship might be the cause of the rejection.

These test statistics have non-standard limiting distributions with no closed form and, therefore, critical values were obtained by resorting to simulation methods. In this paper, we examine types of structural break that were not previously tabulated, which are the change in slope with stable intercept,

$$y_t = \mu + \beta_1' x_t + \beta_2' x_t D_t + u_t, \tag{S}$$

as well as a model with change in slope and no constant term,

$$y_t = \beta_1' x_t + \beta_2' x_t D_t + u_t. \tag{S_{nc}}$$

For proper comparison, and following Gregory and Hansen (1996, p. 110), we obtained critical values for these types of shifts, with a single regressor, using the same response surface: with 10 000 replications for sample dimensions $T = 50, 100, 150, 200, 250$ and 300, critical values at the p percent level are obtained and then the regression

$$C(p,T) = \psi_0 + \psi_1 T^{-1} + \text{error},$$

is run. The critical values at the 5% significance level for the (S) model are -4.685 (*GH-ADF* and *GH-*Z_t tests) and -39.172 (*GH-*Z_α test). For the (S_{nc}) model, the critical values are -4.192 for the *GH-ADF* and *GH-*Z_t tests, and -30.322 for the *GH-*Z_α test, respectively.

2.3 Tests with Cointegration as the Null Hypothesis

The tests described in the previous sections are based on the principle of testing for a unit root in the residuals of the cointegrating regression. Other tests have been developed which test whether the residuals are stationary and, therefore, have cointegration as the null hypothesis. Since we are focusing on the effects of neglected parameter changes, it is also interesting to relate cointegration tests with structural change tests, as the first may be derived from the latter.

Hansen (1992) proposed some LM-type structural change tests in cointegrated models, making use of the Fully-Modified OLS estimator. A versatile feature of those tests is the possibility of using them as cointegration tests. In fact, if the alternative hypothesis is that the intercept follows a random walk, then structural change testing becomes cointegration testing, albeit with the null hypothesis of cointegration. In model (1), if y_t and x_t are not cointegrated, then the error term u_t is integrated of order one. Decomposing u_t such that $u_t = w_t + v_t$, being w_t a random walk and v_t a stationary term, the model then becomes

$$y_t = \alpha_{1t} + \beta' x_t + v_t, \tag{6}$$

with $\alpha_{1t} = \alpha_1 + w_t$, that is, the intercept "absorbs" the random walk w_t when there is no cointegration.

In view of this fact, Hansen (1992) suggested the use of the statistic

$$L_c = \frac{\sum_{t=1}^{T} \hat{s}_t' \hat{M}_t^{-1} \hat{s}_t}{\hat{\omega}_{1.2} T}, \tag{7}$$

to test the null of cointegration, where $\hat{s}_t$represents the scores of the FM-OLS estimates and the weighting matrix $\hat{M}$ is the moments matrix of the regressors. However, this statistic was designed to test the stability of the whole cointegration vector, so there are advantages in regarding a version that tests only (partial) structural change in the intercept. Hao (1996) developed this version, labelling it L_c^0 where the superscript 0 reflects the fact that the test is constructed for testing partial structural change in the intercept. Furthermore, Hao (1996) points out that this version is equivalent to an already known statistic, used by Kwiatkowski, Phillips, Schmidt and Shin (1992) to test for stationarity. Shin (1994), Harris and Inder (1994) and McCabe *et al.* (1997), for example, extend its use to test for the null hypothesis of cointegration (see Gabriel, 2001 for a survey). Here, we use the latter version

$$MLS = T^{-2}\frac{\sum_{t=1}^{T}(\sum_{j=1}^{t}\hat{\varepsilon}_j)^2}{\hat{\sigma}^2}, \tag{8}$$

based on the dynamic OLS estimator of Saikkonen (1991) with filtered residuals $(\hat{\varepsilon}_j)$ from an ARIMA$(p,1,1)$ model, and using the variance estimator $(\hat{\sigma}^2)$ suggested by Leybourne and McCabe (1999) (see McCabe *et al.,* 1997 and Gabriel, 2001 for more details on the computation of the statistic).

It is important, however, to stress that a researcher should be cautious in interpreting the results of these tests, since a rejection does not entail the immediate acceptance of the alternative hypothesis for which they were constructed. For instance, if the MLS statistic rejects, that does not mean that there is no cointegration, since it also has power against parameter instability. The only plausible conclusion one can draw is that the traditional specification of a cointegration model such as (1) (assuming parameter stability) is not supported by the data. The same applies to structural change tests used as cointegration tests.

With this in mind, Hao (1996) proposed a robust test for cointegration, with the objective of overcoming an eventual rejection of the null hypothesis due to a discrete break in the constant term. The transformation may be implemented with the L_c^0 version of (**??**), inserting a dummy variable in the regression that tries to capture the possible break in the intercept. Given that the change point is unknown, the test consists of taking the smallest L_c^0 statistic computed for all possible break dates, that is, the test statistic is $\inf_{\lambda\in J} L_c^0$. The model is now written as

$$y_t = \alpha_{1t} + \alpha_{2t}D_t + \beta' x_t + u_t, \tag{9}$$

with D_t equal to 0 if $t \leq [T\lambda]$ and equal to 1 if $t > [T\lambda]$.

3 Monte Carlo Analysis

In this section, we resort to Monte Carlo simulations to evaluate the finite-sample properties of the cointegration procedures discussed in section 2, when we allow for cointegration with changes in parameters. First, we describe the DGP and the experimental design used in the simulations. This is followed, in the next section, by a discussion of the numerical results.

In our experiments, we consider Markov switching cointegration as defined in Hall *et al.* (1997), where long run parameters switch between different cointegrating regimes. The DGP is specified as

$$\begin{aligned} y_t &= \alpha(s_t) + \beta(s_t)x_t + \sigma(s_t)u_t, \qquad (10) \\ x_t &= x_{t-1} + \nu_t, \qquad t = 1, ..., T, \end{aligned}$$

where y_t and x_t are both scalar, with

$$\alpha(s_t) = \alpha_0 + \alpha_1 s_t, \qquad (11)$$

$$\beta(s_t) = \beta_0 + \beta_1 s_t, \qquad (12)$$

$$\sigma(s_t) = \sigma_0 + \sigma_1 s_t, \qquad (13)$$

where s_t is a binary random variable in $S = \{0, 1\}$, indicating the unobserved regime or state of the cointegrating relationship at date t. It is postulated that $\{s_t\}$ is a stationary first-order Markov chain in S with transition matrix $P = (p_{ij})$, where

$$p_{ij} = \Pr(s_t = j | s_{t-1} = i), \qquad i, j \in S. \qquad (14)$$

Furthermore, it is assumed that $\{s_t\}$ is independent of $\{u_t\}$ and $\{v_t\}$. In this way, the cointegration equation will undergo discrete shifts induced by the values of the Markov chain $\{s_t\}$, with the cointegration vector changing stochastically between $(1, -\alpha_0, -\beta_0)$ and $(1, -\alpha_0 - \alpha_1, -\beta_0 - \beta_1)$, while u_t represents the extent to which the system is out of long run equilibrium. Note that the variance of errors is also allowed to switch between regimes. For simplicity, only a single-regressor model, with no deterministic trends, is considered.

To see what the effects of regime shifts in a cointegrating relationship may be, consider the simpler case where only the intercept is switching,

$$y_t = \alpha_0 + \alpha_1 s_t + \beta x_t + u_t. \qquad (15)$$

If switching is neglected, then the researcher would be estimating $y_t = \mu + \beta x_t + e_t$, where $e_t = \alpha_0 + \alpha_1 s_t - \mu + u_t$. Hence, we see that not accounting for regime switching will introduce further autocorrelation in the errors, as we will show next[1]. In order to derive the autocorrelation function of e_t, we need to restrict $\mu = (\alpha_1 - \alpha_0)\pi + \alpha_0$, where π represents the unconditional probability of staying in regime 1 [2], so that e_t is zero-mean and then autocovariances are derived as $E(e_t e_{t-k}) = (\alpha_1 - \alpha_0)^2 cov(s_t, s_{t-k})$. As pointed out by Nelson *et al.* (2001), the $cov(s_t, s_{t-k})$ may be expressed as $\pi(P(s_t = 1|s_{t-k} = 1) - \pi^2$, which converges geometrically to 0. Therefore, the autocovariance function of e_t also decays geometrically to 0, as does its autocorrelation function $\rho_{e,k}$. This means that, even if u_t is white noise, the switching intercept will generate an autocorrelation pattern in the errors.

To have an idea of the precise effects, let us obtain an expression of the first-order autocorrelation of the new error term. The variance of e_t would be $(\alpha_1 - \alpha_0)^2(\pi - \pi^2) + \sigma_u^2$, maintaining the assumption of independence between $\{s_t\}$ and $\{u_t\}$. Hence,

$$\rho_{e,1} = \frac{(\alpha_1 - \alpha_0)^2(\pi p_{11} - \pi^2)}{(\alpha_1 - \alpha_0)^2(\pi - \pi^2) + \sigma_u^2}. \tag{16}$$

From this expression, we see that the autocorrelation will increase with the shift magnitude, while if a regime is more persistent than the other, the variance of e_t decreases and therefore the autocorrelation is milder. For instance, if $u_t \sim i.i.d.(0, 1)$, $p_{00} = p_{11} = 0.98$ and for a shift of magnitude 4 ($\alpha_1 = 4$), ρ_e equals 0.768, whereas if $p_{00} = 0.98$ and $p_{11} = 0.9$, ρ_e is 0.607. If $\alpha_1 = 1$, then in the first case $\rho_{e,1}$ is considerably smaller, 0.192. Also, notice that contrary to intuition, the more persistent the regimes are (i.e., less shifts occurring), the more autocorrelation they will produce. This in accordance with Diebold and Inoue (2001) and Timmermann (2000), for example, which show that increasing the transition probabilities generates higher autocorrelation for a given process subject to breaks.

Moreover, e_t has also an ARMA representation, as discussed in Nelson *et al.* (2001), with the MA coefficient given by $\theta = p_{00} + p_{11} - 1$, arising from the AR(1) representation of s_t. In principle, autocorrelation-robust tests as the ones studied in this paper could tackle at least part of the problem, although we know from previous literature that difficulties in the tests performance are to be expected when structural breaks occur. Note that if the variance of u_t is regime-dependent, then the denominator of $\rho_{e,1}$ will reflect that as $(\alpha_1 - \alpha_0)^2(\pi - \pi^2) + \pi\sigma_{u1}^2 + (1 - \pi)\sigma_{u0}^2$. Further complications would arise if we allowed for a switching slope or considered more complex autocorrelated processes for u_t. These issues will be investigated in the Monte Carlo simulations.

It should be noticed that we allow for regime shifts under the hypothesis of no cointegration, which was not considered previously. Very seldom in applied work does the researcher takes into

[1] Notice that our case is similar to the one studied by Nelson *et al.* (2001, section 2.1).

[2] Given by $(1 - p_{00)}/(2 - p_{00} - p_{11})$.

account this possibility. Nevertheless, this is in line with the recent research on unit roots and structural breaks reviewed in the introduction. Also, this situation should be considered in order to maintain some symmetry with the case of Markov switching cointegration. Although it adds more complexity to the problem, there is no reason why we should restrict the behaviour under the no cointegration hypothesis to a simple random walk, especially if the marginal process contains breaks itself.

We study the performance of the tests for different types of changes. In a first instance, we analyse the case of shifts occurring in the slope ($\beta(s_t)$) in a model with no intercept ($\alpha_0 = \alpha_1 = 0$). We also consider the case of changing slopes with a stable intercept, as well as changing intercepts with constant slope coefficient ($\beta_0 = \beta_1$). Finally, we study the case were both intercept and slope coefficients switch.

Concerning the magnitude of the breaks in the coefficients, we fix $\alpha_0 = 1$ and $\beta_0 = 1$ for the relevant cases and let α_1 and β_1 take on the values (1, 4) and (0.5, 1, 4), respectively. Other values and combinations are obviously possible, but we believe these values to be empirically plausible. In addition, we also study the situation where the variance of the errors may vary and thus we let $\sigma_0 = 1$, while $\sigma_1 \in \{0.5, 1\}$.

As can be seen, this type of model is very flexible, encompassing the regime-shift models discussed by Gregory and Hansen (1996) when $p_{11} = 1$ or $p_{00} = 1$ (i.e., with an absorbing regime). This specification also allows for a wide range of regime changes, depending on the values of the transition probabilities. In our simulations, the values of the transition probabilities are taken from $(p_{00}, p_{11}) \in \{(0.98, 0.98), (0.95, 0.95), (0.95, 0.9)\}$. We attempt here to experiment with different settings for the p_{ij}'s without neglecting their empirical congruence. The first pair of transition probabilities $(p_{00}, p_{11}) = (0.98, 0.98)$ implies highly persistent, almost absorbing regimes, with very few shifts, each regime persisting on average 50 time periods[3]. The pair $(p_{00}, p_{11}) = (0.95, 0.95)$, on the other hand, is less persistent, with an average regime duration of 20 time periods. While the first two pairs allow for symmetry in the persistence of the states, the $(p_{00}, p_{11}) = (0.95, 0.9)$ implies that the second regime is less likely than regime 0, with a mean duration of 10 time periods, therefore originating a more volatile cointegrating relationship. Other values could be experimented, but the simulations have to be reduced to manageable proportions. Furthermore, these values seem sensible, as we may expect some breaks to occur in a long run relationship, although not very frequently. Again, it should be emphasized that both the number and the location of regime shifts are not specified in this DGP.

[3] The duration of regime i is given by $1/(1 - p_{ii})$.

To have an idea of the differences in performance caused by the presence of regime shifts and variances, a "benchmark" model with no regime switching is also evaluated. For every DGP, the error term u_t is generated as an autoregressive process $u_t = \rho u_{t-1} + \varepsilon_t$, $\varepsilon_t \sim n.i.d.(0,1)$, with $\rho = 0$, $\rho = 0.75$ and $\rho = 1$. The aim is to evaluate the tests properties with different error structures, since in an applied work context the disturbances are likely to be, at least, serially correlated. Note that when $\rho = 1$, this will allows us to obtain an estimate of the size of the null-of-no-cointegration (NNC) tests, while representing the empirical power of the null-of-cointegration (NC) tests. Conversely, when $\rho = 0$ and $\rho = 0.75$, we get the empirical power of NNC tests and size estimates for the NC tests. Additionally, the process ν_t in (10) is generated as $n.i.d.(0,1)$, uncorrelated with u_t. The selected sample dimensions are $T = 100$ and 200. In all experiments, the number of replications is 2500. In order to attenuate the effect of initial values of the random number generator, $50 + T$ observations are generated in each replication (setting $x_1 = 0$), but the first 50 observations are discarded.

Thus, and before proceeding to the next section, perhaps it is useful to summarize the questions we are trying to answer with the simulations outlined above. These experiments will help us to gauge the effects of different shift magnitudes, as well as of switching error variances. Moreover, the asymptotic behaviour of the tests in this context is also considered, along with the effects of a significant degree of correlation. On the other hand, by varying the transition probabilities, we are able to determine the impact of different degrees of persistence in cointegration regimes. Finally, and in the context of our model, we try to isolate and characterize the effects of shifts in each cointegrating coefficient. The results of the simulations are analysed next.

4 Numerical Results

The bulk of the results are shown in the Appendix. Thus, Tables 1 to 11 display estimates of rejection frequencies of the different tests at the 5% level of significance. In parentheses, size-corrected powers are presented for NNC tests, the adjustments being based on the corresponding results with $\rho = 1$ in each table[4]. Given the way the DGP is parameterized, it is not clear which value for ρ should be used (under the null hypothesis of cointegration) to obtain size-adjusted powers for NC tests, so we will abstain from presenting such results for this type of tests.

[4] If one was to consider the hypothesis of no cointegration with no regime shifts, then the adjustments in power should have been made with the results from Table 1. Nominal power does not, however, depend on the specification of this hypothesis.

Table 1 summarizes results from previous literature, namely those in Gregory and Hansen (1996, Table 2), showing the results for the model with no regime switching. The first part of this Table concerns the case of no intercept in the model, with the ADF, *GH-ADF* and Z_α tests rejecting the null of no cointegration more often than they should, although not as severely as the Z_t test. On the other hand, the $GH\text{-}Z_\alpha$ test tends to be biased towards the null, while the $GH\text{-}Z_t$ and *MLS* tests display reasonable Type-I error estimates, at least for $\rho = 0$ in the latter case. In terms of power, standard tests perform better and are less affected by autocorrelation. As for the model with constant term, the situation is similar. We present the three versions of Gregory-Hansen tests and it seems that the version designed for model (S) performs slightly better, in general.

Tables 2 to 11 present the results when the cointegration vector is allowed to switch between different regimes. Instead of discussing the results for each set of experiments, perhaps it is more interesting to highlight some general common features of the simulations output (regardless the particular model under study), which help to answer the questions posed in the end of the previous section. First, it is clear that, independently of other parameter values, as the size of the break increases, both the power and size performance of all tests worsens, as expected (compare Table 2a with Tables 3a and 4a, and so forth). The problem seems to affect standard tests to a larger extent than *GH*-type tests, at least in terms of the ability to find cointegration. Concerning the *MLS* test, it is more affected in terms of significance level distortions than in terms of power, which could be predicted from the results in Lee *et al.* (1997). It should be said, however, that for small breaks ($\beta_1 = 0.5$ and $\alpha_1 = 1$) all tests perform reasonably well.

On the other hand, changes in variance have ambiguous effects (see sections in each Table). A mild increase in the rejection frequencies under the null of NNC tests is also accompanied by slightly higher nominal power, while both power and size distortions decrease for the *MLS* test. If we consider size-adjusted power, we observe that it stays very much the same, with marginal increases. Although this somehow contradicts the results in Hamori and Tokihisa (1997) and Kim *et al.* (2000) for univariate series and single deterministic breaks, it is more in accordance with Nelson *et al.* (2001). Note that, in our case, it is not possible to distinguish between upward shifts or downward shifts in variance (unless only one switch in regime occurs), since the relationship is switching between two states at unknown timings. Therefore, we may expect an "averaging" effect, in terms of types of changes in variances, to be taking place and thus having a not very dramatic impact in the performance of the tests.

Thirdly, increasing the size of the sample does not always have a positive impact on the tests finite-sample abilities, especially when there is no autocorrelation, although significant improvements occur for $\rho = 0.75$. Occasionally, higher powers are attained when the sample size is 100, except for the *MLS* test, again confirming the results in Lee *et al.* (1997). However, it is clear that, in general, the estimated Type-I error probabilities for both types of tests diverge from the nominal value of 5% as T grows, and the tendency is aggravated for larger shifts, quite severely in the case of the *MLS* test with $\rho = 0$. This is not surprising, since, on one hand, we should expect some improvements due to the longer sample length, but, on the other hand, this is contradicted by the fact that the number of breaks will increase, even in the case of more persistent regimes.

Moving next to the combined effects of regime shifts and autocorrelation, it is interesting to notice that the overrejection tendency of the *MLS* test is attenuated when $\rho = 0.75$, while the power of the ADF improves slightly. This may have to do with the fact that these tests are correcting for autocorrelation parametrically (as discussed in sections 2.1 and 2.3) and that the correction is being more effective for this structure of errors correlation[5]. On the other hand, and as expected, autocorrelation in the errors affects the power of the other NNC tests, and *GH* tests to a greater extent than standard tests. Nevertheless, this becomes less problematic as the sample size grows.

Concerning the persistence in cointegration regimes, given by p_{00} and p_{11}, even though the number of breaks is larger when the transition probabilities decrease from 0.98 to 0.95, the degree of autocorrelation is smaller, as conjectured from the autocorrelation function $\rho_{e,1}$ in section 3. Thus, the simulations show that standard tests do a better job at rejecting a false null hypothesis of no cointegration. On the other hand, Gregory-Hansen tests perform better when the p_{ij}'s $(i = j)$ are 0.98, probably because, being robust to a single break, they are able to cope better with the smaller number of shifts. Still, the effects of more breaks become apparent in the excessive frequency of rejections of the null of no cointegration. This is also the case when there is asymmetry in the regimes ($p_{00} = 0.95$, $p_{11} = 0.9$), although power improves, since the autocorrelation function of the residuals is a decreasing function of $|p_{00} - p_{11}|$ (see also Nelson *et al.*, 2001, describing similar implications for the univariate case). As for the KPSS-type test, the converse situation takes place: more breaks produce a slight decrease in the estimated power, while reducing the size distortions when the null of cointegration is true.

Regarding the tests behaviour for different model formulations, it is clear from the results

[5] Indeed, additional experiments not reported here show that if a non-parametric version of the KPSS statistic is used in this context, the effect of autocorrelation increases monotonically, as usual.

of the experiments that regime shifts in the slope coefficient have more negative implications for the performance of all tests. Moreover, allowing the intercept to switch jointly with the slope coefficient has only a marginal impact on the tests performance, when compared with the situation where only the slope is shifting, as may be observed from the comparison of Tables 6-7 with Tables 10-11. In these circumstances, the use of the robust test of Hao (1996) is somewhat preferable relatively to that of the *MLS* test, as the former is less oversized than the first. Notwithstanding this, the size distortions for Hao's (1996) test are still considerable and its power is in general inferior to that of the *MLS* test.

Finally, a word on the tests relative performance. First, as the simulations make clear, Phillips-Ouliaris-type tests are superior to ADF-type tests in terms of (nominal and size-ajusted) power, although more liberal in general. Secondly, there are no considerable advantages in the use of robust tests, especially when autocorrelation in the errors is present. Within this class of tests, the $GH\text{-}Z_\alpha$ version seems to be the most well-balanced in terms of power and size. Turning to NC tests, although their power remains reasonable across DGP's, the problem lies in the excessive number of rejections of the null of cointegration, when the DGP is in fact cointegrated. This evidence suggests that these tests may, in some circumstances, tend too behave as structural change tests rather than cointegration tests, since they also have power against this type of misspecification, as discussed in section 2.3.

5 An Empirical Illustration

To illustrate what the effects of unaccounted stochastic structural breaks on cointegration tests may be, we look at a simple empirical example, using US data on stock prices and dividends[6]. Several studies have focused on present value models of stock prices and dividends, albeit without providing conclusive evidence, possibly because of regime changes. Figure 1 shows the series and it is possible to observe the abrupt changes in the time path of the variables. To overcome this, Driffill and Sola (1998) explain the deviations from stock prices fundamentals by allowing the dividends process, as well as the present value relationship, to switch between two regimes.

Assuming that the series are non-stationary, it is natural to ask whether they are cointegrated or not. However, if the long run relationship suffered regime changes, we may expect difficulties in detecting cointegration, according to the results of our Monte Carlo study. Table 12 reports

[6] The data is taken from Shiller (1989) and updated by the this author. The stock prices are January values for the Standard and Poor Composite Index, from 1900 to 1995, while dividends are year-averages. The series are deflated by January values of the producer price index.

the results from a set of cointegration tests that include the ADF test, the Phillips-Ouliaris tests and Gregory-Hansen tests, as well as DOLS asymptotically efficient estimates[7] (see Saikkonen, 1991) of the cointegrating relationship $y_t = \beta x_t + u_t$, where y_t and x_t represent real stock prices and dividends, respectively. All tests for the null hypothesis of no cointegration fail to reject, whereas the KPSS-type test (*MLS*) of McCabe *et al.* (1997) clearly rejects the existence of a long run (stable) relationship between stock prices and real dividends. Note in particular that Gregory-Hansen tests also fail to indicate the presence of cointegration. Hence, a researcher, using these tools, would find evidence against the existence of cointegration between the variables in this dataset.

Now, assume, without further testing and for expositional simplicity, that the series are cointegrated, although with parameter changes (which is in accordance with the results of Driffill and Sola, 1998). To explicitly account for the possible regime shifts in the relationship, we fit a Markov switching system to the present value relationship and the log of real dividends process,

$$y_t = \beta_i x_t + \theta_i v_t, \qquad v_t \sim N(0,1) \tag{17}$$

$$\log x_t = \mu_i + \log x_{t-1} + \omega_i u_t, \qquad u_t \sim N(0,1) \tag{18}$$

where $i = 0, 1$ for state i, following Driffill and Sola (1998). As we can observe in Table 13, the results are similar to those of Driffill and Sola (1998) in that the means and variances appear to be different across regimes. In the regime 0, we have a low growth/high volatility state in the dividends process, with cointegration vector $(1, -\beta_0)$, $\beta_0 = 19.3636$, while regime 1 corresponds to a high growth/low volatility regime with $(1, -\beta_1)$, $\beta_1 = 30.0884$. The probabilities of staying at each regime are $p_{00} = 0.9798$ for regime 0 and $p_{11} = 0.9843$ for regime 1. These estimates contrast with the results in Table 12 for the "invariant" model, where $\beta = 25.356$, which is approximately the average of the two regimes.

If we take these results as a good approximation of the true model, it would be interesting to assess the performance of the cointegration tests used above in this context. Since the results in Table 12 may be specific to the particular sample considered here, a simple Monte Carlo exercise is undertaken in which the estimated model of Table 13 is taken as the DGP, 2500 replications are generated and each of these is tested for cointegration.

By looking at the results displayed in Table 14, we confirm that both types of tests have serious difficulties in distinguishing between cointegration and no cointegration. The size distortions are considerable (first line for null of no cointegration tests and remaining lines for the *MLS*

[7] The number of leads and lags in the DOLS estimation (corresponding estimates not reported) is 1 and was determined using the AIC criterion.

test), less so for the ADF test, while power is very low, especially if size-corrected. Therefore, these procedures are of little use in terms of providing sensible results in such a situation.

6 Conclusion

In this paper, we have investigated the finite-sample properties of cointegration tests when the cointegration vector is subject to regime shifts. It would be natural to expect the procedures under scrutiny in this paper to have their performance worsened when multiple shifts occur, as they were designed for testing in different environments. Still, it seems relevant to study their behaviour, at least as a starting point for future research.

In our experiments, we have characterized which factors contribute to aggravate the tests behaviour. Indeed, a combination of high regime persistence, large magnitude of shifts and autocorrelation literally destroy the tests ability to detect cointegration, particularly if slope coefficients are responsible for the structural breaks. On the other hand, heteroskedasticity in the equilibrium errors as formulated in this paper have little impact on the performance of the tests.

Recent empirical research shows that it is relevant to consider structural changes in many univariate and multivariate non-stationary time series. Notwithstanding this, an appropriate empirical modelling strategy accounting for structural changes is yet to be defined. This paper sought to contribute further to this discussion.

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7 Appendix

Table 1 - Testing for cointegration with no regime shifts

$\alpha = 0$, $\beta = 1$ (No constant model)						
	$T = 100$			$T = 200$		
$\rho =$	0	0.75	1	0	0.75	1
ADF	0.99 (0.988)	0.934 (0.921)	0.06	1.00 (1.00)	0.997 (0.943)	0.052
Z_α	1.00 (1.00)	0.989 (0.973)	0.072	1.00 (1.00)	1.00 (1.00)	0.066
Z_t	1.00 (1.00)	0.991 (0.95)	0.102	1.00 (1.00)	1.00 (1.00)	0.087
GH-ADF (S_{nc})	0.978 (0.97)	0.741 (0.644)	0.066	0.999 (0.998)	0.975 (0.972)	0.06
GH-Z_α (S_{nc})	1.00 (1.00)	0.728 (0.855)	0.026	1.00 (1.00)	1.00 (1.00)	0.042
GH-Z_t (S_{nc})	1.00 (1.00)	0.728 (0.72)	0.051	1.00 (1.00)	1.00 (1.00)	0.05
MLS	0.044	0.148	0.856	0.046	0.074	0.948
$\alpha = 1$, $\beta = 1$						
ADF	0.98 (0.973)	0.864 (0.778)	0.062	1.00 (1.00)	0.989 (0.984)	0.06
Z_α	1.00 (1.00)	0.949 (0.889)	0.073	1.00 (1.00)	1.00 (1.00)	0.06
Z_t	1.00 (1.00)	0.928 (0.854)	0.078	1.00 (1.00)	1.00 (1.00)	0.065
GH-ADF (S)	0.981 (0.974)	0.588 (0.455)	0.078	0.996 (0.994)	0.971 (0.958)	0.068
GH-ADF (C)	0.985 (0.973)	0.675 (0.453)	0.114	0.998 (0.994)	0.972 (0.937)	0.107
GH-ADF (CS)	0.965 (0.975)	0.476 (0.276)	0.122	0.994 (0.99)	0.938 (0.887)	0.094
GH-Z_α (S)	1.00 (1.00)	0.456 (0.666)	0.023	1.00 (1.00)	0.995 (0.998)	0.039
GH-Z_α (C)	1.00 (1.00)	0.428 (0.602)	0.024	1.00 (1.00)	0.994 (0.994)	0.05
GH-Z_α (CS)	1.00 (1.00)	0.176 (0.381)	0.016	1.00 (1.00)	0.953 (0.967)	0.038
GH-Z_t (S)	1.00 (1.00)	0.584 (0.585)	0.05	1.00 (1.00)	0.993 (0.993)	0.05
GH-Z_t (C)	1.00 (1.00)	0.648 (0.571)	0.067	1.00 (1.00)	0.996 (0.992)	0.072
GH-Z_t (CS)	1.00 (1.00)	0.445 (0.355)	0.068	1.00 (1.00)	0.976 (0.959)	0.068
MLS	0.049	0.173	0.946	0.05	0.094	0.976

Table 2a - Change in slope, no constant ($T = 100$)

$\beta_1 = 0.5,\ \sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95,\ p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.73 (0.715)	0.754 (0.729)	0.06	0.781 (0.739)	0.797 (0.74)	0.069	0.868 (0.821)	0.859 (0.78)	0.079
Z_α	0.928 (0.907)	0.846 (0.788)	0.083	0.958 (0.922)	0.882 (0.782)	0.099	0.979 (0.949)	0.925 (0.82)	0.113
Z_t	0.934 (0.906)	0.833 (0.738)	0.105	0.963 (0.915)	0.878 (0.717)	0.125	0.982 (0.946)	0.926 (0.784)	0.138
GH-ADF	0.909 (0.897)	0.64 (0.556)	0.068	0.863 (0.838)	0.614 (0.521)	0.072	0.888 (0.851)	0.665 (0.545)	0.079
GH-Z_α	0.971 (0.98)	0.678 (0.776)	0.03	0.952 (0.962)	0.636 (0.70)	0.035	0.959 (0.965)	0.68 (0.708)	0.045
GH-Z_t	0.971 (0.97)	0.618 (0.60)	0.055	0.95 (0.946)	0.592 (0.575)	0.055	0.961 (0.949)	0.64 (0.57)	0.077
MLS	0.334	0.231	0.836	0.304	0.21	0.814	0.214	0.178	0.81
$\beta_1 = 0.5,\ \sigma_1 = 0.5$									
ADF	0.754 (0.73)	0.793 (0.751)	0.067	0.813 (0.767)	0.836 (0.757)	0.085	0.891 (0.846)	0.891 (0.80)	0.098
Z_α	0.963 (0.944)	0.887 (0.805)	0.089	0.984 (0.955)	0.914 (0.788)	0.114	0.994 (0.98)	0.952 (0.839)	0.128
Z_t	0.968 (0.946)	0.879 (0.769)	0.11	0.986 (0.956)	0.909 (0.738)	0.135	0.995 (0.977)	0.951 (0.796)	0.168
GH-ADF	0.916 (0.903)	0.664 (0.55)	0.073	0.88 (0.855)	0.644 (0.506)	0.082	0.912 (0.886)	0.691 (0.537)	0.087
GH-Z_α	0.985 (0.988)	0.686 (0.757)	0.036	0.978 (0.981)	0.661 (0.699)	0.042	0.989 (0.989)	0.698 (0.70)	0.05
GH-Z_t	0.986 (0.985)	0.648 (0.634)	0.065	0.975 (0.971)	0.627 (0.582)	0.074	0.987 (0.981)	0.67 (0.578)	0.085
MLS	0.311	0.222	0.819	0.279	0.203	0.799	0.184	0.161	0.784
$\beta_1 = 0.5,\ \theta_1 = 1$									
ADF	0.779 (0.748)	0.814 (0.755)	0.072	0.826 (0.76)	0.85 (0.748)	0.103	0.899 (0.847)	0.90 (0.778)	0.112
Z_α	0.98 (0.966)	0.916 (0.819)	0.108	0.993 (0.968)	0.937 (0.758)	0.148	0.998 (0.986)	0.966 (0.794)	0.174
Z_t	0.983 (0.966)	0.914 (0.769)	0.127	0.995 (0.972)	0.935 (0.719)	0.171	0.999 (0.986)	0.968 (0.76)	0.215
GH-ADF	0.917 (0.902)	0.685 (0.528)	0.086	0.886 (0.847)	0.667 (0.476)	0.102	0.92 (0.884)	0.719 (0.481)	0.109
GH-Z_α	0.99 (0.99)	0.706 (0.706)	0.05	0.987 (0.986)	0.692 (0.65)	0.06	0.994 (0.983)	0.72 (0.636)	0.071
GH-Z_t	0.99 (0.988)	0.671 (0.58)	0.082	0.988 (0.98)	0.663 (0.526)	0.09	0.994 (0.979)	0.701 (0.529)	0.106
MLS	0.287	0.211	0.797	0.248	0.192	0.753	0.158	0.152	0.753

Table 2b - Change in slope, no constant ($T = 200$)

$\beta_1 = 0.5$, $\sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95$, $p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.676 (0.661)	0.846 (0.837)	0.057	0.902 (0.839)	0.95 (0.907)	0.084	0.962 (0.908)	0.979 (0.936)	0.106
Z_α	0.93 (0.886)	0.921 (0.864)	0.097	0.991 (0.96)	0.992 (0.942)	0.132	0.999 (0.986)	0.999 (0.974)	0.17
Z_t	0.928 (0.884)	0.91 (0.846)	0.112	0.99 (0.957)	0.989 (0.924)	0.155	0.999 (0.986)	0.998 (0.965)	0.195
GH-ADF	0.778 (0.766)	0.834 (0.815)	0.058	0.853 (0.779)	0.872 (0.784)	0.095	0.924 (0.856)	0.922 (0.829)	0.118
GH-Z_α	0.953 (0.954)	0.896 (0.89)	0.054	0.982 (0.962)	0.948 (0.90)	0.082	0.996 (0.979)	0.979 (0.921)	0.118
GH-Z_t	0.939 (0.925)	0.845 (0.837)	0.059	0.97 (0.952)	0.901 (0.846)	0.076	0.989 (0.974)	0.96 (0.878)	0.122
MLS	0.65	0.27	0.927	0.425	0.165	0.89	0.264	0.109	0.89
$\beta_1 = 0.5$, $\theta_1 = 0.5$									
ADF	0.713 (0.688)	0.888 (0.875)	0.062	0.906 (0.824)	0.963 (0.909)	0.105	0.969 (0.909)	0.982 (0.949)	0.12
Z_α	0.964 (0.929)	0.956 (0.903)	0.107	0.996 (0.966)	0.994 (0.936)	0.186	1.00 (0.992)	1.00 (0.975)	0.232
Z_t	0.965 (0.926)	0.948 (0.879)	0.132	0.995 (0.968)	0.994 (0.925)	0.218	1.00 (0.992)	0.999 (0.968)	0.255
GH-ADF	0.804 (0.779)	0.871 (0.837)	0.067	0.864 (0.78)	0.908 (0.81)	0.103	0.937 (0.87)	0.949 (0.852)	0.125
GH-Z_α	0.982 (0.978)	0.933 (0.92)	0.06	0.993 (0.979)	0.969 (0.911)	0.096	1.00 (0.991)	0.99 (0.942)	0.116
GH-Z_t	0.976 (0.972)	0.886 (0.876)	0.066	0.989 (0.974)	0.94 (0.868)	0.106	0.998 (0.986)	0.976 (0.901)	0.124
MLS	0.609	0.237	0.921	0.399	0.14	0.876	0.245	0.099	0.87
$\beta_1 = 0.5$, $\theta_1 = 1$									
ADF	0.738 (0.678)	0.912 (0.876)	0.072	0.914 (0.814)	0.971 (0.899)	0.144	0.973 (0.909)	0.988 (0.933)	0.164
Z_α	0.981 (0.95)	0.975 (0.918)	0.146	0.989 (0.972)	0.997 (0.923)	0.301	1.00 (0.992)	1.00 (0.962)	0.36
Z_t	0.982 (0.956)	0.971 (0.904)	0.169	0.998 (0.977)	0.996 (0.911)	0.333	1.00 (0.993)	1.00 (0.953)	0.384
GH-ADF	0.828 (0.787)	0.893 (0.845)	0.084	0.88 (0.77)	0.928 (0.804)	0.137	0.944 (0.868)	0.956 (0.858)	0.152
GH-Z_α	0.989 (0.986)	0.957 (0.924)	0.085	0.997 (0.989)	0.981 (0.911)	0.127	1.00 (0.996)	0.994 (0.94)	0.153
GH-Z_t	0.986 (0.984)	0.923 (0.898)	0.084	0.995 (0.985)	0.96 (0.873)	0.134	1.00 (0.994)	0.988 (0.904)	0.164
MLS	0.572	0.216	0.898	0.375	0.128	0.835	0.223	0.094	0.833

Table 3a - Change in slope, no constant ($T = 100$)

$\beta_1 = 1$, $\sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95$, $p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.62 (0.597)	0.575 (0.546)	0.06	0.678 (0.58)	0.641 (0.521)	0.088	0.795 (0.685)	0.753 (0.602)	0.108
Z_α	0.766 (0.708)	0.712 (0.709)	0.108	0.817 (0.656)	0.77 (0.533)	0.146	0.914 (0.725)	0.876 (0.575)	0.187
Z_t	0.771 (0.695)	0.682 (0.546)	0.115	0.819 (0.652)	0.75 (0.481)	0.162	0.915 (0.72)	0.868 (0.534)	0.213
GH-ADF	0.858 (0.832)	0.61 (0.532)	0.071	0.762 (0.712)	0.571 (0.461)	0.09	0.812 (0.716)	0.64 (0.456)	0.101
GH-Z_α	0.912 (0.916)	0.681 (0.70)	0.046	0.829 (0.817)	0.606 (0.57)	0.062	0.865 (0.818)	0.67 (0.555)	0.083
GH-Z_t	0.896 (0.893)	0.585 (0.566)	0.064	0.81 (0.782)	0.545 (0.46)	0.08	0.853 (0.787)	0.617 (0.457)	0.106
MLS	0.396	0.329	0.808	0.352	0.277	0.755	0.244	0.218	0.736
$\beta_1 = 1$, $\theta_1 = 0.5$									
ADF	0.648 (0.613)	0.629 (0.583)	0.064	0.698 (0.615)	0.692 (0.557)	0.094	0.833 (0.716)	0.796 (0.626)	0.116
Z_α	0.825 (0.773)	0.749 (0.65)	0.107	0.871 (0.735)	0.807 (0.57)	0.154	0.944 (0.815)	0.896 (0.644)	0.178
Z_t	0.827 (0.771)	0.724 (0.604)	0.119	0.877 (0.728)	0.797 (0.521)	0.171	0.946 (0.819)	0.893 (0.608)	0.209
GH-ADF	0.869 (0.845)	0.627 (0.52)	0.076	0.796 (0.749)	0.594 (0.476)	0.091	0.848 (0.782)	0.65 (0.481)	0.097
GH-Z_α	0.935 (0.934)	0.682 (0.679)	0.05	0.879 (0.862)	0.616 (0.586)	0.055	0.915 (0.882)	0.675 (0.57)	0.071
GH-Z_t	0.921 (0.916)	0.597 (0.55)	0.071	0.86 (0.841)	0.566 (0.509)	0.079	0.904 (0.807)	0.628 (0.503)	0.10
MLS	0.395	0.293	0.80	0.34	0.241	0.76	0.236	0.185	0.758
$\beta_1 = 1$, $\sigma_1 = 1$									
ADF	0.664 (0.63)	0.667 (0.607)	0.07	0.722 (0.61)	0.734 (0.568)	0.106	0.844 (0.742)	0.83 (0.658)	0.122
Z_α	0.872 (0.812)	0.789 (0.659)	0.115	0.917 (0.786)	0.845 (0.578)	0.169	0.966 (0.861)	0.922 (0.655)	0.206
Z_t	0.876 (0.815)	0.771 (0.614)	0.131	0.922 (0.792)	0.835 (0.545)	0.195	0.968 (0.863)	0.922 (0.622)	0.237
GH-ADF	0.881 (0.846)	0.639 (0.50)	0.084	0.818 (0.758)	0.62 (0.465)	0.102	0.862 (0.793)	0.669 (0.441)	0.11
GH-Z_α	0.952 (0.95)	0.697 (0.662)	0.058	0.911 (0.893)	0.639 (0.566)	0.067	0.943 (0.918)	0.697 (0.553)	0.077
GH-Z_t	0.943 (0.934)	0.621 (0.525)	0.084	0.898 (0.865)	0.593 (0.468)	0.096	0.938 (0.899)	0.652 (0.449)	0.118
MLS	0.373	0.277	0.794	0.338	0.232	0.744	0.224	0.184	0.732

Table 3b - Change in slope, no constant ($T = 200$)

$\beta_1 = 1,\ \sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95,\ p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.57 (0.506)	0.67 (0.608)	0.067	0.849 (0.628)	0.875 (0.648)	0.151	0.945 (0.734)	0.952 (0.73)	0.202
Z_α	0.758 (0.623)	0.772 (0.61)	0.137	0.956 (0.688)	0.957 (0.65)	0.277	0.991 (0.784)	0.994 (0.711)	0.367
Z_t	0.746 (0.615)	0.742 (0.574)	0.155	0.949 (0.692)	0.948 (0.625)	0.298	0.993 (0.79)	0.992 (0.696)	0.37
GH-ADF	0.681 (0.67)	0.668 (0.654)	0.054	0.779 (0.627)	0.756 (0.56)	0.126	0.89 (0.668)	0.871 (0.573)	0.189
GH-Z_α	0.851 (0.812)	0.796 (0.728)	0.077	0.919 (0.77)	0.88 (0.635)	0.154	0.975 (0.792)	0.958 (0.633)	0.226
GH-Z_t	0.808 (0.80)	0.693 (0.676)	0.075	0.873 (0.761)	0.794 (0.598)	0.145	0.953 (0.981)	0.92 (0.599)	0.216
MLS	0.646	0.428	0.904	0.39	0.265	0.827	0.228	0.168	0.803
$\beta_1 = 1,\ \theta_1 = 0.5$									
ADF	0.60 (0.536)	0.733 (0.681)	0.069	0.865 (0.666)	0.904 (0.718)	0.148	0.949 (0.819)	0.966 (0.83)	0.188
Z_α	0.818 (0.698)	0.83 (0.668)	0.143	0.971 (0.754)	0.974 (0.701)	0.286	0.996 (0.888)	0.996 (0.832)	0.364
Z_t	0.814 (0.694)	0.803 (0.634)	0.16	0.968 (0.77)	0.968 (0.686)	0.307	0.996 (0.89)	0.996 (0.814)	0.374
GH-ADF	0.723 (0.684)	0.719 (0.683)	0.067	0.801 (0.653)	0.80 (0.599)	0.132	0.908 (0.74)	0.895 (0.668)	0.165
GH-Z_α	0.882 (0.852)	0.825 (0.771)	0.081	0.949 (0.845)	0.902 (0.71)	0.152	0.986 (0.888)	0.968 (0.746)	0.192
GH-Z_t	0.857 (0.834)	0.747 (0.706)	0.076	0.906 (0.828)	0.837 (0.656)	0.143	0.976 (0.882)	0.938 (0.71)	0.189
MLS	0.686	0.362	0.906	0.428	0.21	0.834	0.256	0.123	0.823
$\beta_1 = 1,\ \sigma_1 = 1$									
ADF	0.613 (0.536)	0.781 (0.708)	0.083	0.879 (0.655)	0.922 (0.726)	0.165	0.956 (0.815)	0.973 (0.84)	0.201
Z_α	0.871 (0.743)	0.872 (0.702)	0.167	0.98 (0.784)	0.984 (0.705)	0.35	0.998 (0.913)	0.998 (0.85)	0.429
Z_t	0.869 (0.745)	0.858 (0.672)	0.185	0.98 (0.799)	0.978 (0.688)	0.378	0.998 (0.915)	0.998 (0.836)	0.441
GH-ADF	0.743 (0.677)	0.761 (0.687)	0.086	0.814 (0.648)	0.838 (0.628)	0.152	0.919 (0.775)	0.921 (0.726)	0.179
GH-Z_α	0.916 (0.88)	0.852 (0.782)	0.097	0.964 (0.879)	0.925 (0.744)	0.16	0.992 (0.924)	0.976 (0.778)	0.201
GH-Z_t	0.896 (0.868)	0.787 (0.733)	0.086	0.941 (0.872)	0.87 (0.705)	0.164	0.985 (0.918)	0.954 (0.739)	0.20
MLS	0.67	0.332	0.894	0.427	0.192	0.812	0.259	0.118	0.803

Table 4a - Change in slope, no constant ($T = 100$)

$\beta_1 = 4$, $\sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95$, $p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.422 (0.365)	0.369 (0.308)	0.086	0.504 (0.237)	0.473 (0.201)	0.178	0.679 (0.26)	0.65 (0.215)	0.289
Z_α	0.562 (0.319)	0.561 (0.243)	0.252	0.659 (0.168)	0.658 (0.156)	0.422	0.826 (0.194)	0.824 (0.172)	0.568
Z_t	0.52 (0.304)	0.508 (0.244)	0.218	0.636 (0.17)	0.632 (0.148)	0.386	0.813 (0.202)	0.809 (0.174)	0.56
GH-ADF	0.69 (0.577)	0.592 (0.346)	0.141	0.575 (0.311)	0.538 (0.265)	0.182	0.651 (0.297)	0.623 (0.239)	0.25
GH-Z_α	0.778 (0.577)	0.71 (0.332)	0.202	0.624 (0.293)	0.589 (0.232)	0.23	0.702 (0.305)	0.674 (0.24)	0.276
GH-Z_t	0.687 (0.486)	0.572 (0.278)	0.172	0.562 (0.288)	0.518 (0.229)	0.213	0.643 (0.28)	0.61 (0.21)	0.28
MLS	0.382	0.385	0.715	0.317	0.322	0.578	0.23	0.235	0.535
$\beta_1 = 4$, $\sigma_1 = 0.5$									
ADF	0.45 (0.385)	0.395 (0.323)	0.082	0.532 (0.295)	0.488 (0.237)	0.157	0.704 (0.334)	0.665 (0.269)	0.251
Z_α	0.572 (0.356)	0.572 (0.28)	0.223	0.668 (0.203)	0.67 (0.18)	0.367	0.832 (0.254)	0.83 (0.216)	0.488
Z_t	0.531 (0.351)	0.517 (0.284)	0.208	0.644 (0.214)	0.64 (0.176)	0.346	0.82 (0.256)	0.813 (0.203)	0.488
GH-ADF	0.712 (0.588)	0.594 (0.385)	0.124	0.589 (0.363)	0.543 (0.288)	0.168	0.674 (0.362)	0.623 (0.278)	0.219
GH-Z_α	0.789 (0.636)	0.706 (0.394)	0.166	0.646 (0.362)	0.588 (0.277)	0.184	0.724 (0.368)	0.674 (0.267)	0.226
GH-Z_t	0.708 (0.563)	0.572 (0.325)	0.15	0.586 (0.336)	0.519 (0.245)	0.186	0.671 (0.346)	0.611 (0.247)	0.235
MLS	0.389	0.382	0.738	0.322	0.312	0.595	0.226	0.228	0.583
$\beta_1 = 4$, $\sigma_1 = 1$									
ADF	0.478 (0.408)	0.411 (0.331)	0.085	0.554 (0.316)	0.506 (0.247)	0.155	0.723 (0.384)	0.677 (0.302)	0.225
Z_α	0.592 (0.392)	0.587 (0.312)	0.201	0.682 (0.237)	0.683 (0.199)	0.339	0.845 (0.325)	0.836 (0.256)	0.445
Z_t	0.558 (0.379)	0.532 (0.292)	0.198	0.658 (0.243)	0.649 (0.185)	0.328	0.834 (0.326)	0.822 (0.238)	0.453
GH-ADF	0.734 (0.617)	0.592 (0.384)	0.119	0.614 (0.39)	0.545 (0.293)	0.153	0.698 (0.412)	0.623 (0.288)	0.192
GH-Z_α	0.804 (0.68)	0.70 (0.434)	0.148	0.67 (0.414)	0.592 (0.297)	0.169	0.741 (0.454)	0.67 (0.308)	0.20
GH-Z_t	0.74 (0.605)	0.57 (0.331)	0.144	0.612 (0.404)	0.519 (0.278)	0.175	0.693 (0.414)	0.612 (0.27)	0.21
MLS	0.402	0.377	0.743	0.338	0.31	0.624	0.232	0.219	0.615

Table 4b - Change in slope, no constant ($T = 200$)

$\beta_1 = 4$, $\sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95$, $p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.443 (0.251)	0.424 (0.231)	0.15	0.793 (0.219)	0.781 (0.195)	0.422	0.922 (0.24)	0.921 (0.216)	0.561
Z_α	0.586 (0.184)	0.599 (0.188)	0.374	0.91 (0.153)	0.909 (0.152)	0.733	0.982 (0.187)	0.983 (0.183)	0.874
Z_t	0.528 (0.178)	0.544 (0.174)	0.353	0.89 (0.156)	0.892 (0.145)	0.728	0.978 (0.184)	0.979 (0.176)	0.866
GH-ADF	0.525 (0.351)	0.485 (0.313)	0.132	0.648 (0.245)	0.629 (0.228)	0.285	0.817 (0.228)	0.806 (0.203)	0.426
GH-Z_α	0.682 (0.31)	0.67 (0.267)	0.254	0.818 (0.243)	0.811 (0.222)	0.439	0.932 (0.247)	0.931 (0.221)	0.576
GH-Z_t	0.569 (0.298)	0.54 (0.253)	0.202	0.714 (0.248)	0.696 (0.22)	0.381	0.886 (0.239)	0.877 (0.213)	0.538
MLS	0.58	0.565	0.783	0.304	0.299	0.61	0.164	0.158	0.55
$\beta_1 = 4$, $\sigma_1 = 0.5$									
ADF	0.46 (0.294)	0.442 (0.262)	0.069	0.803 (0.283)	0.789 (0.248)	0.373	0.925 (0.304)	0.924 (0.267)	0.487
Z_α	0.592 (0.221)	0.614 (0.223)	0.143	0.911 (0.192)	0.913 (0.188)	0.676	0.983 (0.267)	0.984 (0.246)	0.796
Z_t	0.539 (0.222)	0.558 (0.218)	0.16	0.893 (0.198)	0.897 (0.186)	0.668	0.978 (0.266)	0.98 (0.24)	0.798
GH-ADF	0.55 (0.408)	0.501 (0.364)	0.067	0.672 (0.295)	0.636 (0.268)	0.263	0.827 (0.318)	0.808 (0.285)	0.378
GH-Z_α	0.697 (0.376)	0.679 (0.314)	0.081	0.826 (0.30)	0.816 (0.267)	0.389	0.936 (0.331)	0.935 (0.284)	0.507
GH-Z_t	0.592 (0.364)	0.553 (0.299)	0.076	0.728 (0.293)	0.702 (0.252)	0.339	0.894 (0.335)	0.882 (0.286)	0.476
MLS	0.594	0.534	0.906	0.318	0.299	0.651	0.169	0.158	0.60
$\beta_1 = 4$, $\sigma_1 = 1$									
ADF	0.408 (0.309)	0.463 (0.288)	0.131	0.808 (0.301)	0.801 (0.267)	0.348	0.928 (0.36)	0.928 (0.32)	0.436
Z_α	0.606 (0.247)	0.632 (0.252)	0.315	0.913 (0.231)	0.919 (0.222)	0.643	0.983 (0.319)	0.984 (0.29)	0.758
Z_t	0.557 (0.242)	0.578 (0.234)	0.30	0.900 (0.227)	0.90 (0.206)	0.65	0.982 (0.322)	0.981 (0.282)	0.76
GH-ADF	0.571 (0.442)	0.518 (0.382)	0.106	0.688 (0.328)	0.648 (0.287)	0.238	0.839 (0.373)	0.815 (0.32)	0.349
GH-Z_α	0.714 (0.429)	0.69 (0.358)	0.201	0.84 (0.352)	0.821 (0.296)	0.362	0.944 (0.404)	0.938 (0.331)	0.464
GH-Z_t	0.618 (0.42)	0.566 (0.338)	0.164	0.743 (0.347)	0.712 (0.284)	0.316	0.903 (0.391)	0.887 (0.315)	0.433
MLS	0.61	0.52	0.822	0.325	0.292	0.664	0.179	0.15	0.64

Table 5a - Change in slope, with constant ($\alpha_0 = 1$, $T = 100$)

$\beta_1 = 0.5$, $\sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95$, $p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.607 (0.647)	0.598 (0.555)	0.065	0.694 (0.629)	0.611 (0.491)	0.087	0.792 (0.719)	0.699 (0.538)	0.105
Z_α	0.904 (0.865)	0.758 (0.607)	0.101	0.922 (0.84)	0.781 (0.514)	0.124	0.958 (0.874)	0.849 (0.565)	0.146
Z_t	0.90 (0.877)	0.695 (0.603)	0.081	0.914 (0.861)	0.713 (0.512)	0.098	0.954 (0.894)	0.806 (0.568)	0.116
GH-ADF	0.90 (0.884)	0.502 (0.39)	0.079	0.822 (0.77)	0.477 (0.308)	0.099	0.845 (0.787)	0.533 (0.348)	0.107
GH-Z_α	0.956 (0.961)	0.424 (0.496)	0.033	0.903 (0.91)	0.374 (0.42)	0.042	0.91 (0.911)	0.411 (0.418)	0.048
GH-Z_t	0.962 (0.959)	0.468 (0.433)	0.061	0.922 (0.902)	0.444 (0.36)	0.066	0.929 (0.908)	0.48 (0.379)	0.076
MLS	0.423	0.338	0.747	0.437	0.344	0.713	0.326	0.286	0.694
$\beta_1 = 0.5$, $\sigma_1 = 0.5$									
ADF	0.712 (0.688)	0.653 (0.591)	0.068	0.732 (0.656)	0.666 (0.486)	0.105	0.826 (0.737)	0.75 (0.503)	0.142
Z_α	0.945 (0.918)	0.80 (0.641)	0.11	0.964 (0.885)	0.821 (0.472)	0.169	0.985 (0.92)	0.876 (0.507)	0.216
Z_t	0.948 (0.924)	0.74 (0.62)	0.086	0.962 (0.902)	0.768 (0.47)	0.128	0.982 (0.934)	0.846 (0.498)	0.184
GH-ADF	0.911 (0.889)	0.524 (0.342)	0.096	0.857 (0.812)	0.513 (0.303)	0.114	0.886 (0.82)	0.545 (0.291)	0.145
GH-Z_α	0.972 (0.976)	0.431 (0.474)	0.044	0.944 (0.938)	0.393 (0.357)	0.061	0.956 (0.944)	0.424 (0.344)	0.068
GH-Z_t	0.979 (0.976)	0.495 (0.412)	0.069	0.958 (0.938)	0.472 (0.326)	0.087	0.967 (0.946)	0.50 (0.313)	0.109
MLS	0.386	0.324	0.746	0.389	0.315	0.698	0.287	0.228	0.645
$\beta_1 = 0.5$, $\sigma_1 = 1$									
ADF	0.74 (0.718)	0.688 (0.636)	0.07	0.762 (0.67)	0.714 (0.484)	0.123	0.849 (0.751)	0.777 (0.453)	0.18
Z_α	0.97 (0.943)	0.836 (0.622)	0.128	0.984 (0.932)	0.858 (0.474)	0.20	0.992 (0.953)	0.904 (0.448)	0.286
Z_t	0.971 (0.958)	0.788 (0.623)	0.092	0.984 (0.943)	0.814 (0.479)	0.153	0.991 (0.962)	0.875 (0.448)	0.238
GH-ADF	0.917 (0.888)	0.555 (0.313)	0.112	0.864 (0.808)	0.552 (0.271)	0.142	0.899 (0.834)	0.565 (0.235)	0.177
GH-Z_α	0.985 (0.983)	0.457 (0.355)	0.068	0.97 (0.956)	0.433 (0.327)	0.081	0.981 (0.966)	0.453 (0.281)	0.097
GH-Z_t	0.986 (0.984)	0.52 (0.356)	0.089	0.978 (0.964)	0.512 (0.325)	0.112	0.989 (0.97)	0.528 (0.276)	0.137
MLS	0.346	0.319	0.742	0.364	0.293	0.654	0.252	0.257	0.613

Table 5b - Change in slope, with constant ($\alpha_0 = 1$, $T = 200$)

$\beta_1 = 0.5$, $\sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95$, $p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.564 (0.53)	0.746 (0.712)	0.067	0.778 (0.657)	0.862 (0.742)	0.116	0.903 (0.76)	0.923 (0.785)	0.167
Z_α	0.875 (0.829)	0.85 (0.773)	0.09	0.969 (0.888)	0.961 (0.826)	0.17	0.994 (0.935)	0.992 (0.862)	0.245
Z_t	0.87 (0.839)	0.812 (0.755)	0.076	0.965 (0.901)	0.937 (0.813)	0.136	0.992 (0.94)	0.984 (0.858)	0.215
GH-ADF	0.734 (0.706)	0.758 (0.70)	0.071	0.78 (0.674)	0.784 (0.609)	0.116	0.874 (0.733)	0.861 (0.612)	0.177
GH-Z_α	0.922 (0.909)	0.812 (0.765)	0.066	0.941 (0.90)	0.839 (0.716)	0.104	0.976 (0.916)	0.917 (0.689)	0.169
GH-Z_t	0.919 (0.905)	0.77 (0.72)	0.064	0.934 (0.902)	0.802 (0.694)	0.10	0.976 (0.922)	0.894 (0.671)	0.16
MLS	0.75	0.39	0.86	0.585	0.298	0.81	0.404	0.214	0.784
$\beta_1 = 0.5$, $\sigma_1 = 0.5$									
ADF	0.62 (0.571)	0.81 (0.766)	0.077	0.812 (0.608)	0.897 (0.709)	0.192	0.916 (0.713)	0.946 (0.73)	0.27
Z_α	0.931 (0.878)	0.909 (0.804)	0.12	0.987 (0.881)	0.979 (0.732)	0.278	0.999 (0.942)	0.996 (0.794)	0.372
Z_t	0.925 (0.884)	0.883 (0.781)	0.102	0.984 (0.898)	0.966 (0.729)	0.247	0.998 (0.948)	0.993 (0.78)	0.334
GH-ADF	0.772 (0.728)	0.804 (0.719)	0.086	0.794 (0.67)	0.835 (0.594)	0.17	0.892 (0.745)	0.896 (0.598)	0.239
GH-Z_α	0.958 (0.946)	0.854 (0.795)	0.083	0.971 (0.93)	0.879 (0.685)	0.167	0.992 (0.948)	0.943 (0.638)	0.24
GH-Z_t	0.957 (0.944)	0.825 (0.754)	0.082	0.968 (0.931)	0.85 (0.669)	0.156	0.99 (0.953)	0.926 (0.642)	0.235
MLS	0.722	0.354	0.843	0.574	0.266	0.745	0.387	0.194	0.698
$\beta_1 = 0.5$, $\sigma_1 = 1$									
ADF	0.651 (0.578)	0.846 (0.782)	0.089	0.834 (0.606)	0.913 (0.712)	0.266	0.929 (0.697)	0.958 (0.703)	0.377
Z_α	0.964 (0.913)	0.944 (0.82)	0.157	0.996 (0.905)	0.989 (0.717)	0.388	0.999 (0.954)	0.998 (0.739)	0.512
Z_t	0.964 (0.928)	0.922 (0.827)	0.124	0.995 (0.929)	0.982 (0.727)	0.352	0.999 (0.963)	0.998 (0.722)	0.475
GH-ADF	0.795 (0.746)	0.835 (0.722)	0.102	0.819 (0.683)	0.863 (0.597)	0.206	0.905 (0.749)	0.916 (0.552)	0.315
GH-Z_α	0.979 (0.962)	0.889 (0.766)	0.11	0.988 (0.948)	0.917 (0.666)	0.215	0.998 (0.971)	0.964 (0.609)	0.333
GH-Z_t	0.981 (0.966)	0.863 (0.751)	0.101	0.988 (0.955)	0.892 (0.653)	0.20	0.998 (0.976)	0.95 (0.612)	0.319
MLS	0.686	0.332	0.819	0.541	0.251	0.687	0.356	0.188	0.629

Table 6a - Change in slope, with constant ($\alpha_0 = 1$, $T = 100$)

$\beta_1 = 1$, $\sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95$, $p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.533 (0.519)	0.435 (0.402)	0.06	0.543 (0.441)	0.442 (0.496)	0.105	0.681 (0.491)	0.582 (0.311)	0.152
Z_α	0.722 (0.627)	0.624 (0.419)	0.13	0.729 (0.504)	0.643 (0.287)	0.198	0.847 (0.56)	0.777 (0.296)	0.269
Z_t	0.702 (0.654)	0.533 (0.413)	0.087	0.702 (0.534)	0.557 (0.284)	0.145	0.824 (0.585)	0.716 (0.30)	0.209
GH-ADF	0.838 (0.778)	0.497 (0.317)	0.103	0.707 (0.576)	0.46 (0.218)	0.142	0.743 (0.568)	0.516 (0.224)	0.177
GH-Z_α	0.868 (0.848)	0.477 (0.369)	0.079	0.736 (0.682)	0.379 (0.276)	0.09	0.76 (0.668)	0.432 (0.251)	0.11
GH-Z_t	0.868 (0.844)	0.463 (0.333)	0.086	0.758 (0.67)	0.409 (0.248)	0.104	0.784 (0.662)	0.468 (0.228)	0.139
MLS	0.496	0.415	0.737	0.502	0.413	0.663	0.375	0.332	0.631
$\beta_1 = 1$, $\sigma_1 = 0.5$									
ADF	0.572 (0.543)	0.486 (0.432)	0.069	0.593 (0.472)	0.503 (0.317)	0.114	0.733 (0.566)	0.632 (0.344)	0.167
Z_α	0.786 (0.704)	0.666 (0.462)	0.126	0.801 (0.595)	0.687 (0.318)	0.203	0.895 (0.657)	0.808 (0.303)	0.265
Z_t	0.772 (0.726)	0.588 (0.464)	0.09	0.781 (0.632)	0.611 (0.321)	0.152	0.885 (0.694)	0.757 (0.315)	0.223
GH-ADF	0.857 (0.81)	0.515 (0.316)	0.111	0.752 (0.635)	0.468 (0.216)	0.13	0.793 (0.646)	0.53 (0.207)	0.166
GH-Z_α	0.901 (0.882)	0.462 (0.352)	0.071	0.798 (0.752)	0.388 (0.268)	0.079	0.828 (0.757)	0.421 (0.235)	0.099
GH-Z_t	0.906 (0.886)	0.472 (0.332)	0.089	0.816 (0.753)	0.426 (0.246)	0.104	0.849 (0.758)	0.477 (0.218)	0.138
MLS	0.486	0.398	0.735	0.494	0.398	0.667	0.361	0.318	0.63
$\beta_1 = 1$, $\sigma_1 = 1$									
ADF	0.602 (0.574)	0.532 (0.476)	0.071	0.631 (0.496)	0.558 (0.338)	0.127	0.767 (0.598)	0.675 (0.351)	0.19
Z_α	0.835 (0.769)	0.700 (0.508)	0.136	0.857 (0.682)	0.732 (0.364)	0.223	0.932 (0.766)	0.837 (0.382)	0.308
Z_t	0.828 (0.782)	0.634 (0.482)	0.10	0.849 (0.708)	0.662 (0.34)	0.168	0.925 (0.785)	0.79 (0.364)	0.255
GH-ADF	0.862 (0.818)	0.524 (0.311)	0.124	0.774 (0.69)	0.493 (0.253)	0.152	0.821 (0.716)	0.532 (0.236)	0.188
GH-Z_α	0.922 (0.911)	0.468 (0.384)	0.086	0.842 (0.818)	0.405 (0.302)	0.094	0.88 (0.839)	0.43 (0.271)	0.113
GH-Z_t	0.93 (0.91)	0.486 (0.344)	0.104	0.856 (0.814)	0.456 (0.278)	0.118	0.90 (0.839)	0.495 (0.257)	0.154
MLS	0.468	0.335	0.734	0.48	0.324	0.65	0.34	0.242	0.604

Table 6b - Change in slope, with constant ($\alpha_0 = 1$, $T = 200$)

$\beta_1 = 1$, $\sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95$, $p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.441 (0.377)	0.521 (0.446)	0.075	0.70 (0.447)	0.722 (0.432)	0.198	0.864 (0.479)	0.865 (0.428)	0.304
Z_α	0.672 (0.508)	0.66 (0.465)	0.137	0.888 (0.538)	0.882 (0.453)	0.328	0.974 (0.555)	0.969 (0.419)	0.459
Z_t	0.632 (0.536)	0.593 (0.455)	0.104	0.86 (0.566)	0.834 (0.454)	0.276	0.965 (0.59)	0.953 (0.42)	0.414
GH-ADF	0.63 (0.572)	0.586 (0.503)	0.089	0.687 (0.462)	0.626 (0.355)	0.183	0.815 (0.48)	0.775 (0.326)	0.30
GH-Z_α	0.785 (0.699)	0.684 (0.518)	0.12	0.822 (0.614)	0.72 (0.401)	0.214	0.916 (0.59)	0.859 (0.339)	0.347
GH-Z_t	0.766 (0.709)	0.606 (0.498)	0.095	0.796 (0.612)	0.655 (0.373)	0.183	0.899 (0.61)	0.819 (0.336)	0.316
MLS	0.771	0.516	0.832	0.562	0.38	0.722	0.375	0.28	0.676
$\beta_1 = 1$, $\sigma_1 = 0.5$									
ADF	0.482 (0.406)	0.596 (0.511)	0.085	0.727 (0.436)	0.772 (0.449)	0.227	0.877 (0.533)	0.895 (0.485)	0.334
Z_α	0.74 (0.609)	0.722 (0.53)	0.145	0.918 (0.617)	0.909 (0.509)	0.356	0.984 (0.656)	0.98 (0.464)	0.478
Z_t	0.717 (0.636)	0.661 (0.54)	0.124	0.90 (0.656)	0.878 (0.513)	0.304	0.979 (0.698)	0.966 (0.468)	0.435
GH-ADF	0.662 (0.612)	0.643 (0.556)	0.094	0.706 (0.507)	0.678 (0.404)	0.195	0.837 (0.527)	0.819 (0.345)	0.292
GH-Z_α	0.831 (0.762)	0.722 (0.548)	0.113	0.866 (0.69)	0.761 (0.439)	0.213	0.947 (0.702)	0.883 (0.356)	0.32
GH-Z_t	0.812 (0.768)	0.66 (0.542)	0.098	0.848 (0.697)	0.707 (0.418)	0.193	0.935 (0.711)	0.853 (0.358)	0.307
MLS	0.768	0.483	0.832	0.577	0.359	0.709	0.39	0.25	0.656
$\beta_1 = 1$, $\sigma_1 = 1$									
ADF	0.505 (0.43)	0.652 (0.565)	0.091	0.749 (0.442)	0.808 (0.481)	0.28	0.89 (0.532)	0.912 (0.495)	0.403
Z_α	0.806 (0.668)	0.786 (0.579)	0.168	0.947 (0.658)	0.94 (0.497)	0.429	0.991 (0.784)	0.989 (0.582)	0.551
Z_t	0.789 (0.688)	0.732 (0.57)	0.134	0.933 (0.689)	0.906 (0.50)	0.375	0.987 (0.791)	0.981 (0.544)	0.516
GH-ADF	0.685 (0.63)	0.692 (0.58)	0.109	0.728 (0.536)	0.736 (0.452)	0.223	0.859 (0.613)	0.85 (0.45)	0.336
GH-Z_α	0.87 (0.816)	0.764 (0.628)	0.125	0.90 (0.772)	0.798 (0.506)	0.24	0.968 (0.833)	0.908 (0.51)	0.372
GH-Z_t	0.862 (0.816)	0.711 (0.592)	0.113	0.89 (0.774)	0.751 (0.476)	0.218	0.962 (0.839)	0.884 (0.50)	0.345
MLS	0.806	0.409	0.824	0.639	0.288	0.674	0.446	0.192	0.616

Table 7a - Change in slope, with constant ($\alpha_0 = 1$, $T = 100$)

$\beta_1 = 4$, $\sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95$, $p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.326 (0.255)	0.261 (0.18)	0.096	0.324 (0.133)	0.20 (0.107)	0.186	0.492 (0.137)	0.46 (0.109)	0.326
Z_α	0.494 (0.256)	0.478 (0.121)	0.26	0.526 (0.109)	0.524 (0.088)	0.40	0.705 (0.119)	0.697 (0.095)	0.57
Z_t	0.41 (0.244)	0.385 (0.144)	0.177	0.447 (0.11)	0.434 (0.087)	0.295	0.64 (0.129)	0.63 (0.098)	0.469
GH-ADF	0.648 (0.338)	0.536 (0.134)	0.275	0.50 (0.17)	0.461 (0.112)	0.32	0.572 (0.133)	0.534 (0.086)	0.398
GH-Z_α	0.713 (0.441)	0.592 (0.159)	0.322	0.473 (0.178)	0.426 (0.109)	0.279	0.514 (0.145)	0.475 (0.088)	0.33
GH-Z_t	0.636 (0.352)	0.493 (0.132)	0.281	0.462 (0.169)	0.408 (0.105)	0.277	0.532 (0.14)	0.48 (0.086)	0.354
MLS	0.476	0.491	0.675	0.455	0.452	0.543	0.342	0.351	0.466
$\beta_1 = 4$, $\sigma_1 = 0.5$									
ADF	0.351 (0.281)	0.271 (0.193)	0.096	0.348 (0.155)	0.30 (0.116)	0.172	0.524 (0.179)	0.471 (0.125)	0.289
Z_α	0.504 (0.286)	0.486 (0.163)	0.242	0.539 (0.128)	0.529 (0.104)	0.374	0.723 (0.161)	0.704 (0.121)	0.514
Z_t	0.432 (0.277)	0.39 (0.163)	0.167	0.466 (0.139)	0.414 (0.10)	0.272	0.658 (0.174)	0.638 (0.126)	0.421
GH-ADF	0.67 (0.395)	0.532 (0.152)	0.239	0.523 (0.218)	0.46 (0.141)	0.28	0.592 (0.183)	0.527 (0.108)	0.352
GH-Z_α	0.728 (0.499)	0.572 (0.168)	0.262	0.491 (0.226)	0.421 (0.134)	0.237	0.542 (0.21)	0.469 (0.11)	0.282
GH-Z_t	0.663 (0.397)	0.492 (0.138)	0.235	0.49 (0.223)	0.41 (0.132)	0.244	0.56 (0.208)	0.474 (0.111)	0.309
MLS	0.474	0.476	0.674	0.453	0.449	0.565	0.338	0.337	0.494
$\beta_1 = 4$, $\sigma_1 = 1$									
ADF	0.379 (0.307)	0.285 (0.209)	0.092	0.376 (0.188)	0.316 (0.128)	0.173	0.551 (0.217)	0.487 (0.136)	0.271
Z_α	0.526 (0.309)	0.496 (0.17)	0.201	0.554 (0.149)	0.541 (0.109)	0.339	0.739 (0.207)	0.716 (0.138)	0.478
Z_t	0.463 (0.302)	0.403 (0.168)	0.198	0.487 (0.159)	0.453 (0.10)	0.328	0.68 (0.223)	0.647 (0.139)	0.396
GH-ADF	0.697 (0.449)	0.527 (0.166)	0.119	0.547 (0.249)	0.461 (0.148)	0.153	0.619 (0.251)	0.531 (0.131)	0.31
GH-Z_α	0.743 (0.54)	0.56 (0.176)	0.148	0.518 (0.275)	0.42 (0.151)	0.169	0.576 (0.283)	0.469 (0.139)	0.234
GH-Z_t	0.692 (0.467)	0.493 (0.157)	0.144	0.523 (0.265)	0.406 (0.142)	0.175	0.592 (0.277)	0.478 (0.128)	0.266
MLS	0.514	0.491	0.743	0.49	0.46	0.624	0.372	0.352	0.516

Table 7b - Change in slope, with constant ($\alpha_0 = 1$, $T = 200$)

$\beta_1 = 4$, $\sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95$, $p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.301 (0.163)	0.277 (0.144)	0.152	0.59 (0.124)	0.574 (0.109)	0.45	0.803 (0.123)	0.796 (0.112)	0.657
Z_α	0.433 (0.136)	0.452 (0.136)	0.322	0.789 (0.102)	0.795 (0.098)	0.684	0.95 (0.109)	0.949 (0.101)	0.875
Z_t	0.354 (0.138)	0.362 (0.131)	0.236	0.73 (0.10)	0.73 (0.096)	0.596	0.93 (0.118)	0.927 (0.105)	0.824
GH-ADF	0.458 (0.227)	0.425 (0.188)	0.223	0.525 (0.127)	0.501 (0.116)	0.371	0.697 (0.13)	0.684 (0.115)	0.564
GH-Z_α	0.584 (0.201)	0.563 (0.129)	0.351	0.642 (0.124)	0.622 (0.096)	0.498	0.804 (0.129)	0.796 (0.107)	0.698
GH-Z_t	0.501 (0.211)	0.46 (0.142)	0.28	0.571 (0.13)	0.557 (0.103)	0.433	0.764 (0.127)	0.751 (0.104)	0.646
MLS	0.664	0.639	0.742	0.43	0.429	0.552	0.288	0.282	0.437
$\beta_1 = 4$, $\sigma_1 = 0.5$									
ADF	0.321 (0.188)	0.295 (0.161)	0.141	0.605 (0.139)	0.587 (0.123)	0.423	0.812 (0.166)	0.804 (0.14)	0.605
Z_α	0.444 (0.156)	0.471 (0.153)	0.30	0.796 (0.128)	0.801 (0.124)	0.648	0.951 (0.16)	0.952 (0.141)	0.829
Z_t	0.371 (0.162)	0.377 (0.146)	0.223	0.737 (0.132)	0.74 (0.118)	0.559	0.932 (0.168)	0.932 (0.148)	0.774
GH-ADF	0.478 (0.265)	0.438 (0.212)	0.186	0.544 (0.153)	0.513 (0.132)	0.335	0.717 (0.166)	0.689 (0.137)	0.523
GH-Z_α	0.602 (0.251)	0.572 (0.162)	0.298	0.652 (0.169)	0.629 (0.13)	0.454	0.817 (0.18)	0.802 (0.134)	0.644
GH-Z_t	0.523 (0.256)	0.471 (0.178)	0.24	0.588 (0.16)	0.563 (0.124)	0.385	0.776 (0.177)	0.757 (0.131)	0.598
MLS	0.678	0.617	0.759	0.448	0.423	0.572	0.293	0.273	0.476
$\beta_1 = 4$, $\sigma_1 = 1$									
ADF	0.334 (0.211)	0.315 (0.178)	0.135	0.618 (0.159)	0.598 (0.135)	0.411	0.82 (0.201)	0.816 (0.161)	0.576
Z_α	0.463 (0.184)	0.486 (0.18)	0.315	0.803 (0.153)	0.808 (0.142)	0.643	0.952 (0.199)	0.954 (0.177)	0.795
Z_t	0.393 (0.18)	0.40 (0.162)	0.30	0.745 (0.16)	0.749 (0.134)	0.65	0.936 (0.202)	0.936 (0.164)	0.75
GH-ADF	0.504 (0.316)	0.449 (0.251)	0.106	0.558 (0.188)	0.523 (0.15)	0.238	0.732 (0.224)	0.699 (0.164)	0.495
GH-Z_α	0.622 (0.312)	0.578 (0.209)	0.201	0.67 (0.217)	0.638 (0.158)	0.362	0.834 (0.24)	0.809 (0.167)	0.606
GH-Z_t	0.553 (0.307)	0.49 (0.205)	0.164	0.61 (0.198)	0.571 (0.137)	0.316	0.795 (0.242)	0.767 (0.163)	0.552
MLS	0.712	0.62	0.822	0.508	0.448	0.664	0.335	0.31	0.494

Table 8a - Change in intercept ($\beta_0 = 1$, $T = 100$)

$\alpha_1 = 1$, $\sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95$, $p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.877 (0.864)	0.802 (0.738)	0.067	0.889 (0.877)	0.811 (0.738)	0.07	0.926 (0.913)	0.826 (0.755)	0.071
Z_α	1.00 (1.00)	0.917 (0.863)	0.075	1.00 (1.00)	0.926 (0.85)	0.076	1.00 (1.00)	0.931 (0.859)	0.078
Z_t	1.00 (1.00)	0.893 (0.823)	0.07	1.00 (1.00)	0.903 (0.827)	0.082	1.00 (1.00)	0.908 (0.822)	0.08
GH-ADF	0.967 (0.958)	0.661 (0.443)	0.122	0.949 (0.938)	0.634 (0.423)	0.122	0.954 (0.939)	0.655 (0.392)	0.128
GH-Z_α	1.00 (1.00)	0.415 (0.573)	0.022	1.00 (1.00)	0.391 (0.548)	0.021	1.00 (1.00)	0.393 (0.541)	0.024
GH-Z_t	1.00 (1.00)	0.63 (0.546)	0.075	1.00 (1.00)	0.614 (0.521)	0.075	1.00 (1.00)	0.628 (0.507)	0.079
MLS	0.285	0.258	0.758	0.263	0.249	0.764	0.191	0.239	0.759
$\inf L_c$	0.169	0.313	0.70	0.209	0.325	0.704	0.148	0.30	0.699
$\alpha_1 = 1$, $\sigma_1 = 0.5$									
ADF	0.908 (0.897)	0.809 (0.755)	0.07	0.918 (0.887)	0.818 (0.654)	0.101	0.939 (0.915)	0.842 (0.617)	0.136
Z_α	1.00 (1.00)	0.926 (0.796)	0.097	1.00 (1.00)	0.937 (0.688)	0.151	1.00 (1.00)	0.939 (0.644)	0.196
Z_t	1.00 (1.00)	0.902 (0.797)	0.084	1.00 (1.00)	0.916 (0.684)	0.124	1.00 (1.00)	0.923 (0.664)	0.182
GH-ADF	0.966 (0.949)	0.676 (0.357)	0.144	0.958 (0.933)	0.678 (0.287)	0.174	0.96 (0.931)	0.677 (0.27)	0.209
GH-Z_α	1.00 (1.00)	0.436 (0.45)	0.049	1.00 (1.00)	0.432 (0.425)	0.051	1.00 (1.00)	0.43 (0.344)	0.065
GH-Z_t	1.00 (1.00)	0.656 (0.476)	0.093	1.00 (1.00)	0.649 (0.404)	0.114	1.00 (1.00)	0.657 (0.348)	0.144
MLS	0.226	0.248	0.752	0.201	0.238	0.705	0.146	0.228	0.654
$\inf L_c$	0.127	0.29	0.686	0.148	0.306	0.659	0.106	0.289	0.652
$\alpha_1 = 1$, $\sigma_1 = 1$									
ADF	0.916 (0.905)	0.808 (0.744)	0.073	0.929 (0.895)	0.82 (0.623)	0.119	0.951 (0.915)	0.85 (0.544)	0.177
Z_α	1.00 (1.00)	0.932 (0.738)	0.124	1.00 (1.00)	0.942 (0.646)	0.195	1.00 (1.00)	0.945 (0.531)	0.279
Z_t	1.00 (1.00)	0.907 (0.76)	0.096	1.00 (1.00)	0.92 (0.659)	0.149	1.00 (1.00)	0.927 (0.535)	0.257
GH-ADF	0.964 (0.921)	0.694 (0.208)	0.196	0.958 (0.921)	0.692 (0.224)	0.217	0.963 (0.917)	0.706 (0.218)	0.266
GH-Z_α	1.00 (1.00)	0.464 (0.244)	0.09	1.00 (1.00)	0.466 (0.324)	0.082	1.00 (1.00)	0.472 (0.27)	0.107
GH-Z_t	1.00 (1.00)	0.663 (0.301)	0.14	1.00 (1.00)	0.674 (0.31)	0.154	1.00 (1.00)	0.69 (0.251)	0.198
MLS	0.182	0.25	0.738	0.156	0.247	0.666	0.117	0.229	0.601
$\inf L_c$	0.094	0.281	0.665	0.109	0.289	0.616	0.08	0.277	0.606

Table 8b - Change in intercept ($\beta_0 = 1$, $T = 200$)

$\alpha_1 = 1$, $\sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95$, $p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.942 (0.934)	0.97 (0.966)	0.057	0.977 (0.972)	0.978 (0.974)	0.062	0.991 (0.987)	0.981 (0.974)	0.063
Z_α	1.00 (1.00)	1.00 (1.00)	0.062	1.00 (1.00)	1.00 (1.00)	0.065	1.00 (1.00)	1.00 (1.00)	0.065
Z_t	1.00 (1.00)	1.00 (1.00)	0.061	1.00 (1.00)	1.00 (1.00)	0.07	1.00 (1.00)	1.00 (1.00)	0.071
GH-ADF	0.949 (0.932)	0.969 (0.922)	0.113	0.955 (0.935)	0.968 (0.927)	0.115	0.973 (0.959)	0.966 (0.924)	0.113
GH-Z_α	1.00 (1.00)	0.99 (0.992)	0.046	1.00 (1.00)	0.987 (0.986)	0.053	1.00 (1.00)	0.99 (0.989)	0.056
GH-Z_t	1.00 (1.00)	0.995 (0.986)	0.073	1.00 (1.00)	0.992 (0.981)	0.076	1.00 (1.00)	0.995 (0.986)	0.078
MLS	0.534	0.169	0.89	0.355	0.136	0.883	0.23	0.126	0.88
$\inf L_c$	0.454	0.474	0.554	0.379	0.456	0.544	0.257	0.422	0.54
$\alpha_1 = 1$, $\sigma_1 = 0.5$									
ADF	0.975 (0.958)	0.972 (0.959)	0.072	0.99 (0.944)	0.98 (0.928)	0.175	0.997 (0.949)	0.982 (0.905)	0.248
Z_α	1.00 (1.00)	1.00 (1.00)	0.108	1.00 (1.00)	1.00 (0.991)	0.244	1.00 (1.00)	1.00 (0.977)	0.326
Z_t	1.00 (1.00)	1.00 (1.00)	0.088	1.00 (1.00)	1.00 (0.99)	0.208	1.00 (1.00)	1.00 (0.98)	0.315
GH-ADF	0.97 (0.944)	0.967 (0.90)	0.138	0.969 (0.922)	0.959 (0.806)	0.241	0.98 (0.948)	0.968 (0.749)	0.32
GH-Z_α	1.00 (1.00)	0.99 (0.946)	0.091	1.00 (1.00)	0.992 (0.899)	0.162	1.00 (1.00)	0.993 (0.834)	0.256
GH-Z_t	1.00 (1.00)	0.996 (0.95)	0.109	1.00 (1.00)	0.995 (0.901)	0.196	1.00 (1.00)	0.996 (0.832)	0.299
MLS	0.411	0.162	0.853	0.259	0.139	0.76	0.161	0.128	0.717
$\inf L_c$	0.343	0.446	0.539	0.26	0.418	0.583	0.163	0.403	0.639
$\alpha_1 = 1$, $\sigma_1 = 1$									
ADF	0.984 (0.967)	0.97 (0.947)	0.092	0.994 (0.947)	0.978 (0.895)	0.264	0.999 (0.944)	0.981 (0.849)	0.367
Z_α	1.00 (1.00)	1.00 (1.00)	0.15	1.00 (1.00)	1.00 (0.97)	0.382	1.00 (1.00)	1.00 (0.912)	0.49
Z_t	1.00 (1.00)	1.00 (1.00)	0.126	1.00 (1.00)	1.00 (0.972)	0.331	1.00 (1.00)	1.00 (0.912)	0.462
GH-ADF	0.973 (0.938)	0.956 (0.83)	0.158	0.976 (0.928)	0.958 (0.732)	0.296	0.982 (0.944)	0.968 (0.624)	0.43
GH-Z_α	1.00 (1.00)	0.987 (0.887)	0.129	1.00 (1.00)	0.995 (0.841)	0.242	1.00 (1.00)	0.991 (0.683)	0.379
GH-Z_t	1.00 (1.00)	0.993 (0.894)	0.142	1.00 (1.00)	0.996 (0.843)	0.268	1.00 (1.00)	0.996 (0.708)	0.422
MLS	0.31	0.165	0.832	0.189	0.148	0.697	0.125	0.134	0.63
$\inf L_c$	0.264	0.423	0.534	0.194	0.394	0.635	0.117	0.39	0.691

Table 9a - Change in intercept ($\beta_0 = 1$, $T = 100$)

$\alpha_1 = 4$, $\sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95$, $p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.447 (0.413)	0.442 (0.377)	0.069	0.501 (0.413)	0.489 (0.316)	0.092	0.683 (0.577)	0.648 (0.412)	0.115
Z_α	0.85 (0.757)	0.585 (0.452)	0.09	0.918 (0.727)	0.648 (0.363)	0.124	0.972 (0.843)	0.79 (0.471)	0.168
Z_t	0.881 (0.805)	0.538 (0.414)	0.072	0.937 (0.788)	0.59 (0.336)	0.104	0.98 (0.902)	0.75 (0.456)	0.157
GH-ADF	0.816 (0.764)	0.518 (0.296)	0.135	0.74 (0.608)	0.413 (0.173)	0.166	0.80 (0.627)	0.496 (0.187)	0.199
GH-Z_α	0.907 (0.92)	0.335 (0.391)	0.036	0.808 (0.821)	0.214 (0.231)	0.046	0.851 (0.82)	0.275 (0.239)	0.061
GH-Z_t	0.967 (0.942)	0.472 (0.352)	0.086	0.924 (0.858)	0.362 (0.217)	0.107	0.956 (0.838)	0.435 (0.216)	0.146
MLS	0.60	0.45	0.76	0.605	0.421	0.734	0.452	0.34	0.675
inf L_c	0.369	0.457	0.67	0.508	0.477	0.664	0.396	0.384	0.659
$\alpha_1 = 4$, $\sigma_1 = 0.5$									
ADF	0.517 (0.475)	0.543 (0.458)	0.068	0.563 (0.464)	0.585 (0.356)	0.103	0.748 (0.588)	0.719 (0.417)	0.142
Z_α	0.975 (0.902)	0.692 (0.502)	0.104	0.99 (0.88)	0.743 (0.361)	0.159	0.999 (0.948)	0.855 (0.438)	0.223
Z_t	0.984 (0.955)	0.647 (0.499)	0.081	0.995 (0.939)	0.697 (0.35)	0.128	0.999 (0.978)	0.815 (0.45)	0.195
GH-ADF	0.836 (0.787)	0.563 (0.266)	0.166	0.753 (0.665)	0.492 (0.166)	0.194	0.81 (0.717)	0.541 (0.167)	0.23
GH-Z_α	0.973 (0.972)	0.354 (0.35)	0.051	0.94 (0.93)	0.264 (0.229)	0.061	0.969 (0.946)	0.311 (0.212)	0.082
GH-Z_t	0.993 (0.98)	0.516 (0.314)	0.101	0.985 (0.948)	0.432 (0.214)	0.127	0.997 (0.962)	0.50 (0.208)	0.176
MLS	0.568	0.412	0.75	0.583	0.378	0.684	0.428	0.313	0.65
inf L_c	0.354	0.422	0.674	0.476	0.444	0.641	0.362	0.361	0.629
$\alpha_1 = 4$, $\sigma_1 = 1$									
ADF	0.572 (0.532)	0.617 (0.534)	0.072	0.625 (0.48)	0.647 (0.406)	0.117	0.789 (0.621)	0.763 (0.398)	0.18
Z_α	0.998 (0.968)	0.775 (0.516)	0.123	0.999 (0.961)	0.818 (0.379)	0.204	0.999 (0.983)	0.89 (0.388)	0.287
Z_t	0.999 (0.988)	0.732 (0.545)	0.097	0.999 (0.979)	0.776 (0.362)	0.151	0.999 (0.996)	0.862 (0.39)	0.26
GH-ADF	0.853 (0.783)	0.611 (0.17)	0.20	0.782 (0.676)	0.566 (0.144)	0.218	0.844 (0.753)	0.604 (0.146)	0.271
GH-Z_α	0.996 (0.976)	0.39 (0.185)	0.088	0.984 (0.965)	0.318 (0.198)	0.085	0.994 (0.979)	0.356 (0.169)	0.113
GH-Z_t	0.999 (0.986)	0.563 (0.222)	0.138	0.998 (0.976)	0.515 (0.201)	0.154	0.999 (0.986)	0.566 (0.164)	0.205
MLS	0.539	0.382	0.734	0.551	0.352	0.658	0.397	0.292	0.598
inf L_c	0.334	0.392	0.666	0.45	0.408	0.622	0.325	0.335	0.594

Table 9b - Change in intercept ($\beta_0 = 1$, $T = 200$)

$\alpha_1 = 4$, $\sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95$, $p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.422 (0.375)	0.789 (0.743)	0.07	0.795 (0.658)	0.912 (0.82)	0.109	0.926 (0.809)	0.959 (0.879)	0.149
Z_α	0.999 (0.989)	0.913 (0.832)	0.08	1.00 (1.00)	0.988 (0.918)	0.132	1.00 (1.00)	0.998 (0.959)	0.174
Z_t	0.999 (0.997)	0.884 (0.81)	0.075	1.00 (1.00)	0.981 (0.905)	0.121	1.00 (1.00)	0.998 (0.956)	0.168
GH-ADF	0.626 (0.513)	0.756 (0.583)	0.123	0.726 (0.469)	0.819 (0.513)	0.18	0.876 (0.648)	0.902 (0.636)	0.216
GH-Z_α	0.995 (0.993)	0.722 (0.692)	0.064	0.998 (0.995)	0.782 (0.642)	0.096	1.00 (1.00)	0.901 (0.702)	0.141
GH-Z_t	0.999 (0.997)	0.762 (0.677)	0.082	1.00 (0.998)	0.835 (0.628)	0.13	1.00 (1.00)	0.932 (0.703)	0.18
MLS	0.884	0.471	0.865	0.724	0.326	0.817	0.504	0.214	0.784
$\inf L_c$	0.781	0.782	0.545	0.787	0.728	0.548	0.609	0.582	0.561
$\alpha_1 = 4$, $\sigma_1 = 0.5$									
ADF	0.488 (0.405)	0.854 (0.798)	0.078	0.81 (0.576)	0.925 (0.776)	0.189	0.937 (0.71)	0.965 (0.796)	0.263
Z_α	1.00 (1.00)	0.973 (0.889)	0.108	1.00 (1.00)	0.998 (0.82)	0.269	1.00 (1.00)	0.999 (0.842)	0.352
Z_t	1.00 (1.00)	0.963 (0.882)	0.088	1.00 (1.00)	0.997 (0.826)	0.226	1.00 (1.00)	0.999 (0.855)	0.33
GH-ADF	0.69 (0.566)	0.844 (0.622)	0.134	0.78 (0.512)	0.887 (0.498)	0.25	0.902 (0.623)	0.934 (0.495)	0.342
GH-Z_α	1.00 (1.00)	0.83 (0.677)	0.098	1.00 (1.00)	0.875 (0.565)	0.167	1.00 (1.00)	0.95 (0.558)	0.272
GH-Z_t	1.00 (1.00)	0.866 (0.70)	0.112	1.00 (1.00)	0.916 (0.567)	0.206	1.00 (1.00)	0.968 (0.556)	0.317
MLS	0.871	0.41	0.852	0.723	0.286	0.738	0.506	0.198	0.689
$\inf L_c$	0.753	0.748	0.545	0.76	0.673	0.594	0.569	0.547	0.651
$\alpha_1 = 4$, $\sigma_1 = 1$									
ADF	0.551 (0.452)	0.868 (0.811)	0.091	0.831 (0.551)	0.933 (0.762)	0.269	0.939 (0.663)	0.968 (0.743)	0.369
Z_α	1.00 (1.00)	0.992 (0.90)	0.154	1.00 (1.00)	0.999 (0.778)	0.402	1.00 (1.00)	0.999 (0.747)	0.492
Z_t	1.00 (1.00)	0.986 (0.906)	0.124	1.00 (1.00)	0.999 (0.795)	0.349	1.00 (1.00)	0.999 (0.753)	0.464
GH-ADF	0.724 (0.591)	0.888 (0.626)	0.155	0.807 (0.54)	0.912 (0.46)	0.306	0.90 (0.663)	0.949 (0.434)	0.428
GH-Z_α	1.00 (1.00)	0.896 (0.642)	0.132	1.00 (1.00)	0.936 (0.572)	0.237	1.00 (1.00)	0.97 (0.487)	0.386
GH-Z_t	1.00 (1.00)	0.922 (0.669)	0.144	1.00 (1.00)	0.957 (0.576)	0.273	1.00 (1.00)	0.98 (0.506)	0.434
MLS	0.853	0.364	0.828	0.698	0.26	0.692	0.474	0.182	0.618
$\inf L_c$	0.732	0.679	0.547	0.722	0.614	0.636	0.512	0.501	0.686

Table 10a - Change in slope and in the intercept ($T = 100$)

$\alpha_1 = \beta_1 = 1$, $\sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95$, $p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.531 (0.509)	0.425 (0.39)	0.062	0.541 (0.43)	0.44 (0.28)	0.107	0.685 (0.488)	0.573 (0.302)	0.154
Z_α	0.719 (0.626)	0.617 (0.42)	0.135	0.733 (0.506)	0.64 (0.273)	0.203	0.849 (0.545)	0.775 (0.286)	0.272
Z_t	0.698 (0.642)	0.529 (0.403)	0.103	0.703 (0.532)	0.555 (0.282)	0.161	0.826 (0.59)	0.71 (0.30)	0.233
GH-ADF	0.856 (0.776)	0.524 (0.27)	0.159	0.73 (0.544)	0.474 (0.178)	0.193	0.77 (0.528)	0.538 (0.166)	0.256
GH-Z_α	0.86 (0.838)	0.362 (0.286)	0.07	0.683 (0.63)	0.271 (0.19)	0.077	0.697 (0.619)	0.296 (0.203)	0.092
GH-Z_t	0.888 (0.846)	0.49 (0.284)	0.13	0.769 (0.638)	0.42 (0.185)	0.151	0.803 (0.627)	0.474 (0.189)	0.201
MLS	0.498	0.422	0.736	0.50	0.405	0.66	0.372	0.332	0.634
$\inf L_c$	0.34	0.392	0.668	0.384	0.395	0.62	0.291	0.334	0.60
$\alpha_1 = \beta_1 = 1$, $\sigma_1 = 0.5$									
ADF	0.566 (0.528)	0.483 (0.424)	0.069	0.588 (0.468)	0.50 (0.311)	0.118	0.728 (0.561)	0.632 (0.343)	0.167
Z_α	0.788 (0.704)	0.659 (0.454)	0.128	0.80 (0.589)	0.688 (0.316)	0.209	0.896 (0.665)	0.804 (0.327)	0.268
Z_t	0.78 (0.719)	0.591 (0.438)	0.104	0.781 (0.614)	0.609 (0.298)	0.171	0.882 (0.693)	0.752 (0.319)	0.244
GH-ADF	0.876 (0.802)	0.526 (0.21)	0.182	0.772 (0.605)	0.482 (0.157)	0.208	0.817 (0.628)	0.544 (0.15)	0.238
GH-Z_α	0.894 (0.877)	0.336 (0.254)	0.076	0.754 (0.722)	0.263 (0.193)	0.073	0.782 (0.728)	0.283 (0.188)	0.085
GH-Z_t	0.918 (0.818)	0.484 (0.222)	0.141	0.832 (0.712)	0.444 (0.168)	0.158	0.876 (0.731)	0.487 (0.171)	0.19
MLS	0.49	0.406	0.732	0.491	0.388	0.66	0.347	0.32	0.624
$\inf L_c$	0.33	0.389	0.659	0.374	0.386	0.62	0.276	0.336	0.606
$\alpha_1 = \beta_1 = 1$, $\sigma_1 = 1$									
ADF	0.600 (0.572)	0.529 (0.473)	0.069	0.623 (0.49)	0.557 (0.337)	0.13	0.765 (0.592)	0.671 (0.345)	0.188
Z_α	0.835 (0.763)	0.698 (0.489)	0.135	0.856 (0.668)	0.73 (0.326)	0.229	0.936 (0.749)	0.834 (0.336)	0.307
Z_t	0.826 (0.786)	0.632 (0.481)	0.109	0.846 (0.696)	0.662 (0.324)	0.19	0.924 (0.769)	0.784 (0.316)	0.274
GH-ADF	0.888 (0.796)	0.548 (0.139)	0.21	0.803 (0.631)	0.511 (0.137)	0.23	0.846 (0.68)	0.553 (0.134)	0.258
GH-Z_α	0.917 (0.882)	0.346 (0.139)	0.105	0.81 (0.762)	0.286 (0.171)	0.084	0.856 (0.795)	0.286 (0.162)	0.094
GH-Z_t	0.945 (0.888)	0.504 (0.153)	0.168	0.882 (0.769)	0.471 (0.161)	0.176	0.916 (0.801)	0.51 (0.166)	0.206
MLS	0.465	0.385	0.735	0.476	0.37	0.649	0.336	0.307	0.607
$\inf L_c$	0.313	0.372	0.662	0.358	0.374	0.602	0.264	0.332	0.591

Table 10b - Change in slope and in the intercept ($T = 200$)

$\alpha_1 = \beta_1 = 1,\ \sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95,\ p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.438 (0.379)	0.521 (0.451)	0.074	0.70 (0.447)	0.723 (0.437)	0.202	0.86 (0.477)	0.862 (0.419)	0.307
Z_α	0.662 (0.517)	0.658 (0.464)	0.139	0.888 (0.537)	0.881 (0.451)	0.332	0.973 (0.564)	0.968 (0.426)	0.462
Z_t	0.629 (0.527)	0.591 (0.446)	0.115	0.858 (0.572)	0.838 (0.456)	0.29	0.964 (0.595)	0.955 (0.425)	0.432
GH-ADF	0.651 (0.538)	0.608 (0.443)	0.138	0.707 (0.445)	0.65 (0.31)	0.251	0.831 (0.446)	0.789 (0.263)	0.381
GH-Z_α	0.768 (0.68)	0.622 (0.438)	0.124	0.775 (0.573)	0.627 (0.332)	0.202	0.878 (0.555)	0.775 (0.277)	0.327
GH-Z_t	0.782 (0.682)	0.61 (0.414)	0.128	0.808 (0.591)	0.66 (0.326)	0.233	0.904 (0.572)	0.813 (0.278)	0.383
MLS	0.772	0.517	0.828	0.566	0.384	0.726	0.377	0.282	0.665
$\inf L_c$	0.678	0.612	0.547	0.646	0.619	0.615	0.524	0.532	0.676
$\alpha_1 = \beta_1 = 1,\ \sigma_1 = 0.5$									
ADF	0.476 (0.397)	0.599 (0.512)	0.083	0.731 (0.424)	0.767 (0.441)	0.228	0.881 (0.531)	0.895 (0.486)	0.334
Z_α	0.739 (0.593)	0.726 (0.519)	0.148	0.919 (0.588)	0.913 (0.463)	0.354	0.984 (0.695)	0.98 (0.528)	0.477
Z_t	0.714 (0.603)	0.664 (0.486)	0.124	0.899 (0.622)	0.877 (0.462)	0.315	0.98 (0.72)	0.968 (0.528)	0.448
GH-ADF	0.687 (0.574)	0.656 (0.483)	0.147	0.737 (0.473)	0.69 (0.327)	0.261	0.85 (0.548)	0.824 (0.33)	0.377
GH-Z_α	0.822 (0.751)	0.668 (0.482)	0.118	0.836 (0.663)	0.666 (0.36)	0.198	0.921 (0.706)	0.809 (0.341)	0.301
GH-Z_t	0.838 (0.761)	0.662 (0.476)	0.138	0.865 (0.678)	0.708 (0.351)	0.248	0.937 (0.716)	0.846 (0.331)	0.374
MLS	0.774	0.484	0.83	0.587	0.357	0.712	0.398	0.247	0.66
$\inf L_c$	0.704	0.629	0.548	0.644	0.622	0.63	0.511	0.52	0.68
$\alpha_1 = \beta_1 = 1,\ \sigma_1 = 1$									
ADF	0.505 (0.42)	0.648 (0.561)	0.092	0.746 (0.436)	0.811 (0.476)	0.283	0.89 (0.53)	0.912 (0.489)	0.404
Z_α	0.798 (0.658)	0.783 (0.554)	0.171	0.945 (0.638)	0.939 (0.469)	0.434	0.991 (0.738)	0.988 (0.509)	0.555
Z_t	0.787 (0.676)	0.728 (0.54)	0.141	0.929 (0.671)	0.905 (0.465)	0.395	0.987 (0.768)	0.981 (0.51)	0.526
GH-ADF	0.716 (0.593)	0.703 (0.478)	0.148	0.752 (0.472)	0.745 (0.316)	0.301	0.866 (0.578)	0.856 (0.316)	0.429
GH-Z_α	0.868 (0.782)	0.708 (0.462)	0.133	0.884 (0.728)	0.712 (0.371)	0.228	0.946 (0.784)	0.839 (0.363)	0.344
GH-Z_t	0.885 (0.795)	0.716 (0.439)	0.148	0.909 (0.729)	0.763 (0.342)	0.282	0.963 (0.794)	0.876 (0.335)	0.42
MLS	0.766	0.46	0.817	0.586	0.34	0.679	0.406	0.238	0.617
$\inf L_c$	0.706	0.632	0.543	0.639	0.607	0.636	0.491	0.516	0.689

Table 11a - Change in slope and in the intercept ($T = 100$)

$\alpha_1 = \beta_1 = 4$, $\sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95$, $p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.324 (0.25)	0.261 (0.181)	0.099	0.322 (0.129)	0.288 (0.105)	0.184	0.493 (0.137)	0.458 (0.105)	0.324
Z_α	0.485 (0.255)	0.469 (0.119)	0.26	0.522 (0.106)	0.523 (0.088)	0.395	0.702 (0.117)	0.698 (0.091)	0.567
Z_t	0.408 (0.246)	0.379 (0.142)	0.196	0.438 (0.111)	0.425 (0.092)	0.317	0.635 (0.121)	0.624 (0.089)	0.493
GH-ADF	0.70 (0.331)	0.587 (0.119)	0.364	0.523 (0.144)	0.49 (0.09)	0.385	0.584 (0.129)	0.56 (0.078)	0.473
GH-Z_α	0.699 (0.427)	0.554 (0.14)	0.33	0.406 (0.156)	0.352 (0.097)	0.25	0.428 (0.134)	0.37 (0.076)	0.282
GH-Z_t	0.679 (0.338)	0.55 (0.115)	0.35	0.486 (0.142)	0.43 (0.088)	0.33	0.535 (0.128)	0.495 (0.077)	0.405
MLS	0.50	0.524	0.677	0.49	0.469	0.55	0.37	0.365	0.461
$\inf L_c$	0.33	0.385	0.565	0.384	0.392	0.485	0.306	0.314	0.424
$\alpha_1 = \beta_1 = 4$, $\sigma_1 = 0.5$									
ADF	0.349 (0.281)	0.275 (0.194)	0.096	0.353 (0.157)	0.298 (0.111)	0.173	0.519 (0.179)	0.471 (0.124)	0.292
Z_α	0.499 (0.283)	0.476 (0.138)	0.238	0.536 (0.129)	0.528 (0.099)	0.37	0.715 (0.16)	0.708 (0.119)	0.518
Z_t	0.433 (0.277)	0.387 (0.158)	0.179	0.454 (0.127)	0.433 (0.097)	0.294	0.65 (0.166)	0.632 (0.109)	0.449
GH-ADF	0.715 (0.365)	0.57 (0.118)	0.328	0.548 (0.189)	0.49 (0.106)	0.363	0.602 (0.167)	0.553 (0.091)	0.433
GH-Z_α	0.717 (0.469)	0.527 (0.14)	0.287	0.43 (0.20)	0.345 (0.112)	0.222	0.461 (0.177)	0.359 (0.087)	0.244
GH-Z_t	0.70 (0.376)	0.539 (0.115)	0.315	0.514 (0.199)	0.431 (0.107)	0.306	0.569 (0.188)	0.488 (0.094)	0.368
MLS	0.508	0.517	0.684	0.494	0.466	0.572	0.375	0.361	0.489
$\inf L_c$	0.329	0.391	0.583	0.388	0.393	0.508	0.302	0.316	0.461
$\alpha_1 = \beta_1 = 4$, $\sigma_1 = 1$									
ADF	0.374 (0.305)	0.288 (0.208)	0.092	0.375 (0.183)	0.316 (0.123)	0.175	0.551 (0.222)	0.485 (0.142)	0.272
Z_α	0.514 (0.306)	0.486 (0.16)	0.215	0.553 (0.15)	0.537 (0.111)	0.348	0.704 (0.204)	0.722 (0.141)	0.482
Z_t	0.458 (0.306)	0.402 (0.177)	0.159	0.478 (0.153)	0.446 (0.108)	0.285	0.678 (0.214)	0.644 (0.13)	0.42
GH-ADF	0.73 (0.40)	0.561 (0.114)	0.32	0.574 (0.21)	0.488 (0.109)	0.344	0.634 (0.204)	0.547 (0.095)	0.386
GH-Z_α	0.73 (0.497)	0.504 (0.132)	0.252	0.457 (0.24)	0.337 (0.198)	0.199	0.487 (0.234)	0.348 (0.098)	0.208
GH-Z_t	0.731 (0.414)	0.527 (0.106)	0.29	0.543 (0.23)	0.43 (0.107)	0.285	0.608 (0.238)	0.484 (0.103)	0.32
MLS	0.517	0.51	0.692	0.491	0.352	0.582	0.368	0.354	0.515
$\inf L_c$	0.34	0.393	0.594	0.387	0.394	0.518	0.306	0.321	0.491

Table 11b - Change in slope and in the intercept ($T = 200$)

$\alpha_1 = \beta_1 = 4$, $\sigma_1 = 0$	$p_{00} = p_{11} = 0.98$			$p_{00} = p_{11} = 0.95$			$p_{00} = 0.95$, $p_{11} = 0.9$		
$\rho =$	0	0.75	1	0	0.75	1	0	0.75	1
ADF	0.302 (0.157)	0.283 (0.141)	0.153	0.59 (0.116)	0.574 (0.102)	0.45	0.805 (0.125)	0.799 (0.112)	0.659
Z_α	0.438 (0.135)	0.455 (0.132)	0.318	0.791 (0.098)	0.796 (0.095)	0.69	0.949 (0.106)	0.949 (0.101)	0.874
Z_t	0.353 (0.153)	0.363 (0.138)	0.25	0.728 (0.096)	0.734 (0.09)	0.613	0.93 (0.112)	0.93 (0.104)	0.832
GH-ADF	0.48 (0.202)	0.448 (0.152)	0.29	0.556 (0.095)	0.54 (0.08)	0.457	0.719 (0.107)	0.708 (0.092)	0.644
GH-Z_α	0.528 (0.202)	0.507 (0.124)	0.339	0.563 (0.097)	0.546 (0.081)	0.454	0.74 (0.103)	0.725 (0.086)	0.654
GH-Z_t	0.507 (0.194)	0.468 (0.124)	0.325	0.586 (0.099)	0.563 (0.083)	0.473	0.769 (0.111)	0.754 (0.088)	0.695
MLS	0.70	0.653	0.752	0.488	0.459	0.551	0.322	0.32	0.433
$\inf L_c$	0.534	0.532	0.581	0.604	0.592	0.651	0.527	0.522	0.621
$\alpha_1 = \beta_1 = 4$, $\sigma_1 = 0.5$									
ADF	0.319 (0.183)	0.296 (0.158)	0.145	0.605 (0.139)	0.588 (0.116)	0.423	0.814 (0.17)	0.805 (0.142)	0.609
Z_α	0.446 (0.162)	0.468 (0.156)	0.295	0.795 (0.122)	0.804 (0.118)	0.65	0.949 (0.159)	0.951 (0.143)	0.83
Z_t	0.371 (0.162)	0.378 (0.146)	0.233	0.734 (0.136)	0.739 (0.12)	0.576	0.934 (0.163)	0.934 (0.142)	0.786
GH-ADF	0.503 (0.256)	0.452 (0.19)	0.266	0.568 (0.12)	0.548 (0.094)	0.423	0.734 (0.159)	0.713 (0.126)	0.606
GH-Z_α	0.55 (0.237)	0.512 (0.142)	0.297	0.583 (0.131)	0.548 (0.096)	0.418	0.755 (0.156)	0.727 (0.112)	0.616
GH-Z_t	0.536 (0.23)	0.478 (0.134)	0.295	0.572 (0.134)	0.572 (0.099)	0.448	0.782 (0.16)	0.761 (0.113)	0.659
MLS	0.712	0.635	0.761	0.495	0.461	0.571	0.323	0.312	0.474
$\inf L_c$	0.549	0.532	0.579	0.609	0.596	0.664	0.529	0.528	0.662
$\alpha_1 = \beta_1 = 4$, $\sigma_1 = 1$									
ADF	0.334 (0.209)	0.313 (0.181)	0.137	0.622 (0.162)	0.603 (0.127)	0.412	0.819 (0.203)	0.814 (0.16)	0.58
Z_α	0.461 (0.184)	0.488 (0.177)	0.283	0.801 (0.15)	0.811 (0.132)	0.632	0.952 (0.194)	0.955 (0.17)	0.80
Z_t	0.388 (0.185)	0.398 (0.165)	0.22	0.742 (0.162)	0.749 (0.137)	0.563	0.937 (0.206)	0.938 (0.171)	0.762
GH-ADF	0.527 (0.283)	0.469 (0.204)	0.244	0.589 (0.154)	0.555 (0.117)	0.405	0.747 (0.195)	0.724 (0.139)	0.586
GH-Z_α	0.58 (0.287)	0.522 (0.168)	0.284	0.603 (0.167)	0.556 (0.115)	0.386	0.773 (0.222)	0.731 (0.147)	0.578
GH-Z_t	0.572 (0.27)	0.49 (0.151)	0.278	0.62 (0.174)	0.579 (0.114)	0.42	0.801 (0.217)	0.767 (0.139)	0.628
MLS	0.725	0.613	0.765	0.509	0.449	0.59	0.332	0.305	0.486
$\inf L_c$	0.57	0.542	0.58	0.617	0.594	0.665	0.533	0.532	0.68

Table 12 - Cointegration Analysis

Tests	ADF	Z_α	Z_t	$GH\text{-}ADF$	$GH\text{-}Z_\alpha$	$GH\text{-}Z_t$	MLS
	-2.117	-10.597	-2.022	-3.20	-20.842	-3.217	7.901^{**}
Estimated β (standard error):				25.353	(0.695)		
Regression standard error:				0.1514			

Table 13 - Markov switching cointegration results

Eq. (**17**)	β_0	β_1	θ_0	θ_1	p_{00}	p_{11}
	19.3636 (0.5795)	30.0884 (0.8339)	0.1466 (0.0192)	0.2995 (0.0635)	0.9798 (0.0376)	0.9843 (0.0422)
Eq. (**18**)	μ_0	μ_1	ω_0	ω_1		
	−0.0041 (0.0095)	0.0316 (0.0041)	0.1513 (0.0193)	0.0462 (0.0092)		

Note: standard errors in brackets

Table 14 - Simulated size and power, model (17)-(18) as the DGP

ρ	ADF	Z_α	Z_t	*GH-ADF*	*GH-*Z_α	*GH-*Z_t	*MLS*
1	0.091	0.214	0.197	0.306	0.411	0.324	0.634
0	0.248 (0.176)	0.346 (0.159)	0.337 (0.16)	0.526 (0.206)	0.62 (0.301)	0.519 (0.215)	0.421
0.75	0.228 (0.15)	0.348 (0.058)	0.329 (0.08)	0.479 (0.124)	0.596 (0.162)	0.472 (0.117)	0.449

Note: Size-adjusted power in parentheses, based on corresponding critical values from $\rho = 1$

Figure 1: Stock Prices and Dividends